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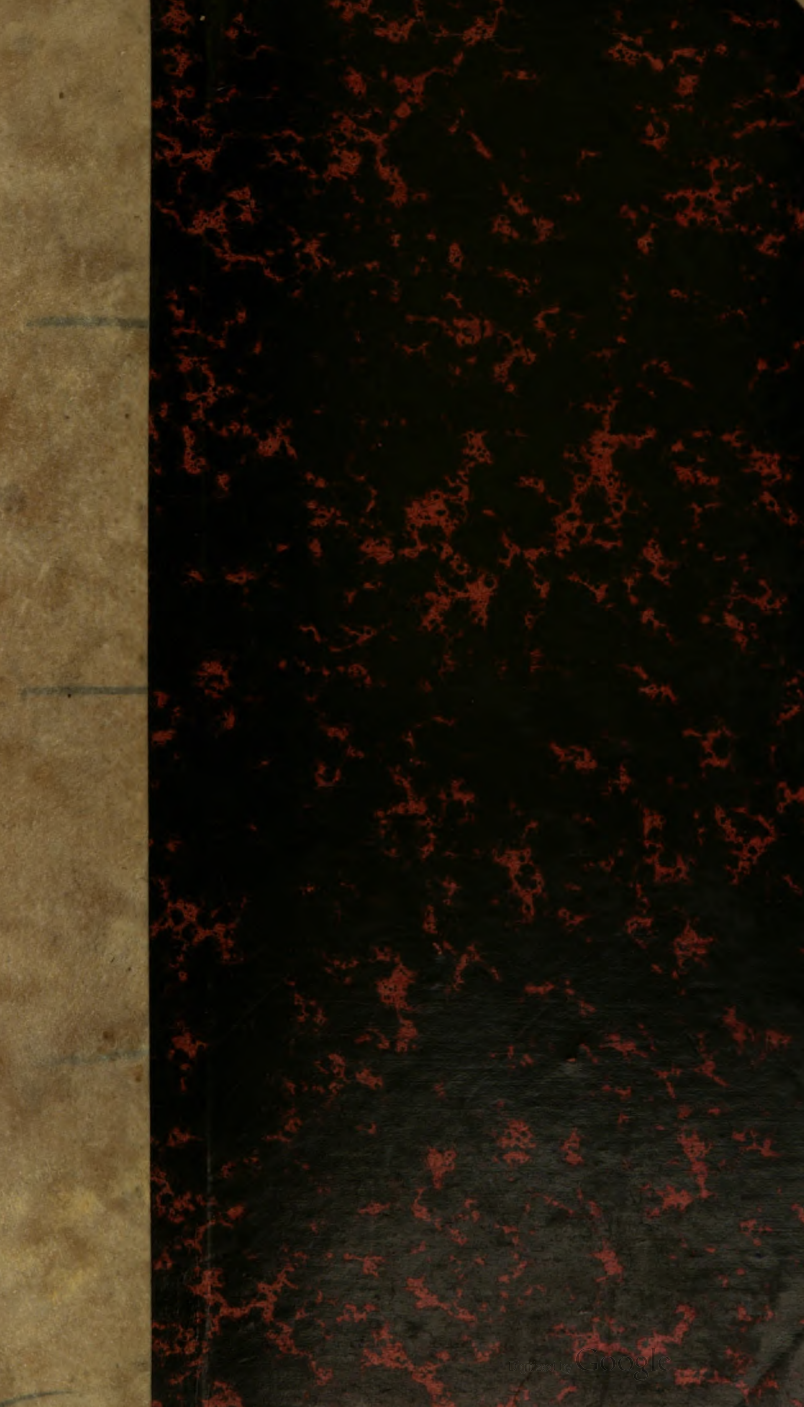
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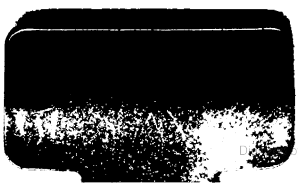
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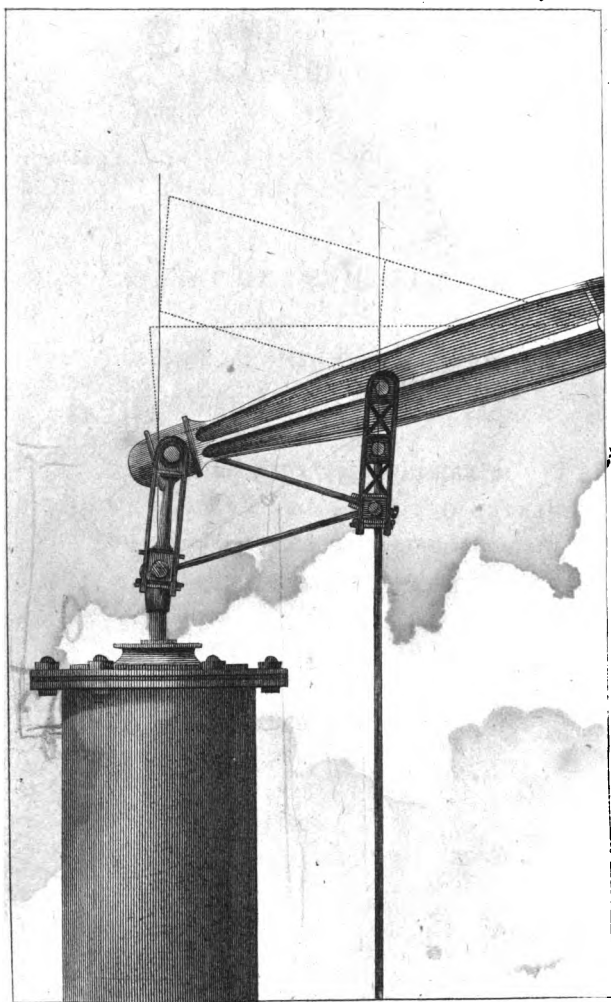
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PARALLEL MOTION.



A
COMPENDIUM OF MECHANICS,

OR

TEXT BOOK,

FOR

**ENGINEERS, MILL-WRIGHTS, MACHINE-MAKERS,
FOUNDERS, SMITHS, &c.**

CONTAINING

PRACTICAL RULES AND TABLES

CONNECTED WITH THE

**STEAM ENGINE, WATER WHEEL, FORCE PUMP,
AND MECHANICS IN GENERAL:**

ALSO,

Examples for each Rule,

CALCULATED IN COMMON DECIMAL ARITHMETIC,

Which renders this Treatise particularly adapted for the use of

OPERATIVE MECHANICS.

BY ROBERT BRUNTON.

WITH PLATES.

GLASGOW :

JOHN NIVEN, JUN., 118, Trongate

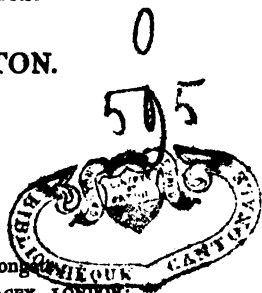
G. & W. B. WHITTAKER, AND KNIGHT & LACEY, LONDON;

BEILBY & KNOTTS, BIRMINGHAM;

OLIVER & BOYD, A. CONSTABLE & CO. AND E. WEST & CO.

EDINBURGH.

1824.



J. Niven, Printer.

TO THE
MECHANICS OF GLASGOW,
THIS COMPENDIUM
OF
PRACTICAL MECHANICS,

IS INSCRIBED BY

THEIR OBEDIENT SERVANT,

Robert Brunton.

PREFACE.

THE following Compilation is submitted to the Mechanics of Glasgow, by one of their number, who hopes it will be found a simple and easy Introduction to the knowledge of Calculations connected with Mechanics.

Most of the Rules and Tables have been selected from the latest eminent Publications on these subjects, and information procured from every possible source, with a view of rendering this Work useful for practical purposes.

The want of a Text Book for Operative Mechanics has been long felt.—The great inconvenience arising from this, was the cause of the Compiler collecting the following Rules for his own personal use:—and having, with several other Mechanics, experienced the great advantage derived from these Memoranda, is induced to submit them to the Public, trusting they will be found to contain much useful information.

GLASGOW,
February, 1824. }

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EXPLANATIONS

OF THE

Characters used in the following Calculations.

-
- + Signifies Addition, as $5 + 3$ is 8.
 — ——— Subtraction, as $5 - 3$ is 2.
 × ——— Multiplication, as 5×3 is 15.
 ÷ ——— Division, as $15 \div 3$ is 5, or $\frac{15}{3}$ is 5.
 :: ——— Proportion, as 2 is to 3, as 4 is to 6.
 = ——— Equality, as $5 + 3 = 8$.
 √ ——— Square Root, as $\sqrt{9} = 3$.
 $\sqrt[3]{}$ ——— Cube Root, as $\sqrt[3]{27} = 3$.

3^2 Signifies that 3 is to be squared as $3^2 = 9$.

3^3 ——— 3 is to be cubed as $3^3 = 27$.

The Bar signifies that 2 numbers are to be taken together, as $3 \times \overline{5 + 3} = 24$.

COMPENDIUM OF MECHANICS.

WEIGHTS AND MEASURES.

TROY WEIGHT.

<i>Gr.</i>	<i>Dwt.</i>	<i>Oz.</i>	<i>Lib.</i>	<i>Grains,</i>	<small>Mark.</small> <i>Gr.</i>
24 =	1			Penny-weights,	<i>Dwt.</i>
480 =	20	1		Ounce,	<i>Oz.</i>
5760 =	240 =	12 =	1	Pound,	<i>Lib.</i>

By this Weight are weighed Gold, Silver, and Jewels.

AVOIRDUPOIS WEIGHT.

<i>Dr.</i>	<i>Oz.</i>	<i>Lib.</i>	<i>Qr.</i>	<i>Cwt.</i>	<i>Ton.</i>	<i>Dram, . .</i>	<small>Mark.</small> <i>Dr.</i>
16 =	1					Ounce,	<i>Oz.</i>
256 =	16 =	1				Pound,	<i>Lib.</i>
7168 =	448 =	28 =	1			Quarter,	<i>Qr.</i>
28672 =	1792 =	112 =	4 =	1		Hund. wt.	<i>Cwt.</i>
573440 =	35840 =	2240 =	80 =	20 =	1	Ton,	<i>Ton</i>

By this Weight all Metals, save Gold and Silver, are weighed.

B

AVOIRDUPOIS WEIGHT.

Oz. Dwt. Gr.

Note.	1 Lib. Avoir.	=	14 . 11 . 15½	Troy.
	1 Oz. do.	=	18 . 5½	do.
	1 Dr. do.	=	1 . 3½	do.

LONG MEASURE.

<i>In.</i>	<i>Ft.</i>				<i>Mark.</i>
				Inch, . . .	<i>In.</i>
12=	1	Yd.		Foot, . . .	<i>Ft.</i>
36=	3 =	1	<i>Pl.</i>	Yard, . . .	<i>Yd.</i>
198=	16½=	5½=	1 <i>Fur.</i>	Pole or Rod, <i>Pl.</i>	
7920=	660 =	220 =	40=1 <i>M.</i>	Furlong, . .	<i>Fur.</i>
63360=	5280 =	1760 =	320=8=1	Mile, . . .	<i>M.</i>

8 Miles = 1 League, marked *Lea.*69 $\frac{1}{13}$ Miles nearly = 1 Degree, marked °

SQUARE MEASURE.

<i>Sq. In.</i>	<i>Sq. Ft.</i>				<i>Mark.</i>
				Square Inch, <i>Sq. In.</i>	
144=	1	<i>Sq. Yd.</i>	 Foot, <i>Sq. Ft.</i>	
1296=	9 =	1	<i>Sq. Pl.</i> Yard, <i>Sq. Yd.</i>	
39204=	272½=	30½=	1 <i>Rd.</i> Pole, <i>Sq. Pl.</i>	
1568160=	10890 =	1210 =	40=1 <i>Acr.</i>	Rood, . . .	<i>Rd.</i>
6272640=	43560 =	4840 =	160=4=1	Acre, . . .	<i>Acr.</i>

1089 Scotch Acres = 1369 English Acres.

DRY MEASURE.

<i>Pts.</i>	<i>Gal.</i>				<i>Mark.</i>	
					Pints, <i>Pts.</i>	
8 =	1	<i>Pec.</i>			Gallon, <i>Gal.</i>	
16 =	2 =	1	<i>Bu.</i>		Peck, <i>Pec.</i>	
64 =	8 =	4	1 <i>Qr.</i>		Bushel, <i>Bu.</i>	
512 =	64 =	32 =	8 =	1 <i>Wey</i>	Quarter, <i>Qr.</i>	
2560 =	320 =	160 =	40 =	5 =	1 <i>Last</i>	Wey, Load, or Ton, <i>Wey</i>
5120 =	640 =	320 =	80 =	10 =	2 =	1 <i>Last</i>

A Chaldron of Coals in London = 36 Bushels, and weighs 3136 lbs. Avoirdupois, or 1 Ton 8 Cwt. nearly.

ALE MEASURE.

<i>Pts.</i>	<i>Qt.</i>				<i>Mark.</i>	
					Pints, <i>Pts.</i>	
2 =	1	<i>Gal.</i>			Quart, <i>Qt.</i>	
8 =	4 =	1	<i>Bar.</i>		Gallon, <i>Gal.</i>	
288 =	144 =	36 =	1	<i>Hhd.</i>	Barrel, <i>Bar.</i>	
432 =	216 =	54 =	1½ =	1 <i>Butt</i>	Hogshead, <i>Hhd.</i>	
864 =	432 =	108 =	3 =	2 =	1 <i>Tun</i>	Butt, <i>Butt.</i>
1728 =	864 =	216 =	6 =	4 =	2 =	1 <i>Tun</i>

Note. The Ale Gallon contains 282 Cubic or Solid Inches.

WINE MEASURE.

<i>pts</i>	<i>qt</i>								Mark
2=	1	<i>gal</i>							Pints, . . . <i>pts</i>
8=	4=	1	<i>tier</i>						Quart, . . . <i>qt</i>
336=	168=	42=	1	<i>hhd</i>					Gallon, . . <i>gal</i>
504=	252=	63=	1½=	1	<i>pun</i>				Tierce, . . <i>tier</i>
672=	336=	84=	2 = 1½=	1	<i>pipe</i>				Hogshead, <i>hhd</i>
1008=	504=	126=	3 = 2 = 1½=	1	<i>tun</i>				Puncheon, . <i>pun</i>
2016=	1008=	252=	6 = 4 = 3 = 2=	1					Pipe or Butt, <i>pipe</i>
									Tun, <i>tun</i>

Note. The Wine Gallon contains 231 Cubic or Solid Inches; and it is remarkable, that the Wine Gallon is to the Ale Gallon, nearly as the Pound Troy is to the Pound Avoirdupois.

SOLID MEASURE.

<i>Cubic Inches</i>	<i>Cub. Ft.</i>				
1728	=	1	<i>Cub. Yd.</i>		
15552	=	9 = 1	<i>Fathom</i>		
373248	=	216 = 8 = 1			

In taking the solid contents of any Mass, there is seldom any other Measure than the Cubic Foot used.

*The old and new French Weights and Measures,
reduced to the English Standard.**

The Paris pound, *poids de marc of Charlemagne*, contains 9216 Paris grains; it is divided into 16 ounces, each ounce into 8 gros (or drams), and each gros into 72 grains; it is equal to 7561 English troy grains.

The English troy pound of 12 ounces, contains 5760 English troy grains, and is equal to 7021 Paris grains.

The English avoirdupois pound of 16 ounces, contains 7000 English troy grains, and is equal to 8538 Paris grains.

To reduce Paris grains to English troy grains, divide by 1.2189.

To reduce Paris ounces to English troy, divide by 1.015734; or the conversion may be made by means of the following Tables.

I. To reduce French to English Troy Weight.

	English Troy Grains.
The Paris Pound	= 7561
Ounce	= 472.5625
Gros	= 59.0703
Grain	= .8204

* See the Technical Repository, Vol. 3, No. 6, for June 1823.



II. To reduce Paris Long Measure to English.

	Eng. Inches.
The Paris Royal Foot of 12 Inches	= 12.7977
The Inch	= 1.0659
The Line, or one-twelfth of an Inch	= .0074

III. To reduce French Cubic Measure to English.

	Eng. Cubical Feet.
The Paris Cubic Foot	= 1.211273
The Cubic Inch	= .000700

IV. Measure of Capacity.

The Paris pint contains 58.145 English cubical inches, and the English Wine pint contains 28.875 cubical inches; or the Paris pint contains 2.0171082 English pints; therefore, to reduce the Paris pint to the English, multiply by 2.0171082.

TABLE of the New French Weights and Measures reduced to the English Standard.

The French *metre*, according to the *Journal de Physique, An. 7. Prair, & Fruct*, is equal to 3 feet, 11.296 lines French, and the *gramme* to 18.827 grains. The *metre* is the ten-millionth part of the distance from the Pole to the Equator. The *gramme* is the weight of a cubic centimetre of water. The French *toise* was 76.734 inches English; and 576 French grains were equal to 472.5 English.—*See Phil. Transact.* vol. 58, p. 326.

MEASURES OF LENGTH.

	English Inches.
Millimetre =03937
Centimetre =39370
Decimetre =	3.93702
Metre =	39.37023
Decametre =	393.70226
Hecatometre =	3937.02260
Chiliometre =	39370.22601
Myriometre =	393702.26014

	M.	P.	Y.	Ft.	In.
A Decametre is =	0	0	10	2	9.7
A Hecatometre =	0	0	109	1	1.0
A Chiliometre =	0	4	213	1	10.2
A Myriometre =	6	1	156	0	.6

4 Furlong

Eight Chiliometres are nearly 5 English miles.

MEASURES OF CAPACITY.

	English Cubic Inches.
Millilitre =06102
Centilitre =61024
Decilitre =	6.10244
Litre =	61.10244
Decalitre =	610.24429
Hecalitre =	6102.44288
Chilolitre =	61024.42878
Myriolitre =	610244.28778

A Litre is nearly $2\frac{1}{8}$ Wine pints.

14 Decilitres are nearly 3 Wine pints.

A Chilolitre is a tun, 12.75 Wine gallons.

WEIGHTS.

	English Grains.
Milligramme =	.0154
Centigramme =	.1544
Décigramme =	1.5444
Gramme =	15.4440
Decagramme =	154.4402
Hecatogramme =	1544.4023
Chiliogramme (Kilogram) =	15444.0234
Myriogramme =	154440.2344

A Decagramme is 6 dwts. 10.44 gr. tr.; or 5.65 dr. avoir.

A Hecatogramme is 3 oz. 8.5 dr. avoir.

A Chiliogramme is 2 lbs. 3 oz. 5 dr. avoir.

A Myriogramme is 22 — 1.15 oz. avoir.

100 Myriogrammes are 1 Ton, wanting 32.8 lbs.

AGRARIAN MEASURES.

Are, 1 square Decametre . = 3.95 Perches.

Hectare = 2 Acres, 1 Rood,
30.1 Perches.

FIR WOOD.

Decistre, 1-10th Stere . = 3.5315 cub. ft. Eng.

Stere, 1 Cubic Metre . = 35.3150 cub. ft.

MENSURATION.

Areas or Surfaces.

PROBLEM I.

To find the area of any Parallelogram, whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid,—See Plate 1, Fig. 1, 2, 3, 4,

RULE. Multiply the length by the perpendicular breadth or height, and the product will be the area.

PROBLEM II.

To find the area of a Triangle.

RULE. Multiply the base by the perpendicular height, and take half the product for the area.

PROBLEM III.

To find the area of a Trapezoid.—See Plate 1. Fig. 5.

RULE. Add together the two parallel sides; then multiply their sum by the perpendicular breadth, or the distance between them, and take half the product for the area.

PROBLEM IV.

To find the area of any Trapezium.—See Fig. 6.

RULE. Divide the Trapezium into two triangles by a diagonal; then find the areas of the triangles by Prob. 2. and add them together for the area of the Trapezium.

PROBLEM V.

To find the area of an Irregular Polygon.—See Fig. 7.

RULE. Draw Diagonals dividing the proposed Polygon into Trapeziums and Triangles; then find the areas of all these separately, and add them together for the contents of the whole Polygon.

PROBLEM VI.

To find the area of a Regular Polygon.

RULE 1. Multiply the perimeter of the Polygon, or sum of its sides, by the perpendicular drawn from its centre on one of its sides, and take half the product for the area.

RULE 2. Square the side of the Polygon; then multiply that square by the tabular area set against its name in the following Table, and the product will be the area.

No. of sides.	NAMES.	AREAS.
3	Trigon, or Triangle .	0.4330127
4	Tetragon, or Square .	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

PROBLEM VII.

To find the diameter and circumference of any circle, the one from the other.—See Fig. 8.

This may be done by either of the three following proportions, viz. As 7 is to 22, so is the diameter to the circumference; or, As 1 is to 3.1416, so is the diameter to the circumference; or, As 113 is to 355, so is the diameter to the circumference.

PROBLEM VIII.

To find the length of any arc of a circle.—See Fig. 9.

RULE. Multiply the decimal .01745 by the degrees in the given arc, and the product by the radius of the circle, for the length of the arc.

PROBLEM IX.

To find the Area of a Circle.

RULE 1. Multiply half the circumference by half the diameter, and the product is the area.

* **RULE 2.** Square the diameter, and multiply that square by the decimal .7854 for the area.

RULE 3. Square the circumference, and multiply that square by the decimal .07958.

PROBLEM X.

* *To find the area of a circular ring, or of the space included between the circumferences of two circles; the one being contained within the other.*

RULE. Take the difference between the areas of the two circles for the area of the ring.

PROBLEM XI.

To find the area of the Sector of a Circle.—See Fig. 10.

RULE 1. Multiply the Radius, or half the diameter, by half the arc of the Sector, for the area; or multiply the whole diameter by the whole arc of the sector, and take $\frac{1}{4}$ of the product.

RULE 2. Compute the area of the whole circle; then say, as 360° is to the degrees in the arc of the sector, so is the area of the whole circle, to the area of the sector.

PROBLEM XII.

* *To find the area of a Segment of a Circle.—See Fig. 11.*

RULE 1. Find the area of the sector, having the same arc with the segment, by the 2nd rule of last

Problem. Find also the area of the triangle, formed by the cord of the segment and the two radii of the sector; then add these together for the answer, when the segment is greater than a semicircle; or subtract them, when it is less than a semicircle: As is evident,

RULE 2. Divide the height of the segment by the diameter, and find the quotient in the column of heights in the following Table. Take out the corresponding area in the next column on the right hand; and multiply it by the square of the circle's diameter for the area of the segment.

When the quotient is not found exactly in the Table, proportion may be made between the next less and greater area, in the same manner as is done with any other Table.

Table of the area of circular Segments.

Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.
.01	.00133	.11	.04701	.21	.11990	.31	.20738	.41	.30319
.02	.00375	.12	.05339	.22	.12811	.32	.21667	.42	.31304
.03	.00687	.13	.06000	.23	.13646	.33	.22603	.43	.32293
.04	.01054	.14	.06683	.24	.14494	.34	.23547	.44	.33284
.05	.01468	.15	.07387	.25	.15354	.35	.24498	.45	.34278
.06	.01924	.16	.08111	.26	.16226	.36	.25455	.46	.35274
.07	.02417	.17	.08853	.27	.17109	.37	.26418	.47	.36272
.08	.02944	.18	.09613	.28	.18002	.38	.27386	.48	.37270
.09	.03502	.19	.10390	.29	.18905	.39	.28359	.49	.38270
.10	.04088	.20	.11182	.30	.19817	.40	.29337	.50	.39270

C

PROBLEM XIII.

To measure long irregular Figures.

RULE. Take or measure the breadth at both ends, and at several places, at equal distances; then add together all these intermediate breadths, and half the two extremes; which sum multiply by the length, and divide by the number of parts for the area. If the perpendiculars or breadths be not at equal distances, compute all the parts separately, as so many trapezoids, and add them all together for the whole area.

PROBLEM XIV.

To find the area of an Ellipsis or Oval.—See Fig. 12.

RULE. Multiply the longest diameter by the shortest; then multiply the product by the decimal .7854, for the area.

PROBLEM XV.

To find the area of an Elliptic Segment.

RULE 1. Find the area of a corresponding circular segment, having the same height, and the same vertical axis or diameter; then say, as the said vertical axis is to the other axis, parallel to the segment's base; so is the area of the circular segment before found, to the area of the elliptic segment sought.

RULE 2. Divide the height of the segment by the vertical axis of the ellipse, and find in the Table of circular segments, Prob. 12, the circular segment having the above quotient for its versed sine; then multiply altogether, this segment and the two axes of the ellipse.

PROBLEM XVI.

To find the area of a Parabola, or its Segment.

See Fig. 13.

✕ **RULE.** Multiply the base by the perpendicular height; then take two-thirds of the product for the area.

SOLIDS.

By Mensuration of Solids, are determined the spaces included by contiguous surfaces; and the sum of the measures of these including surfaces, is the whole surface or superficies of the body.

The measure of a body is called its solidity, capacity, or content.

Solids are measured by cubes, whose sides are inches, feet, or yards, &c.; and hence the solidity of a body is said to be so many cubic inches, feet, yards, &c. as will fill its capacity or space, or another of an equal magnitude.

The least solid measure is the cubic inch; other cubes being taken from it, according to the propor-

tion in the following Table, which is formed by cubing the linear proportions.

Table of Cubes or Solids.

1728	Cubic Inches	make	1	Cubic Foot.
27	Cubic Feet	—	1	Cubic Yard.
166 $\frac{2}{3}$	Cubic Yards	—	1	Cubic Pole.
64000	Cubic Poles	—	1	Cubic Furlong.
512	Cubic Furlongs	—	1	Cubic Mile.

PROBLEM I.

To find the Superficies of a Prism or Cylinder.—See Fig. 14 and 15.

RULE. Multiply the perimeter of one end of the prism, by the length of the solid, and the product will be the surface of all its sides. To which add also the area of the two ends of the prism, when required.

Or, compute the areas of all the sides and ends separately, and add them all together.

PROBLEM II.

To find the surface of a Pyramid or Cone.—See Fig. 16 and 17.

RULE. Multiply the perimeter of the base by the slant height, or length of the side, and half the product will be the surface of the sides, or the sum of

the areas of all the triangles which form it. To which add the area of the end or base, if required.

PROBLEM III.

To find the surface of the Frustum of a Pyramid or Cone, being the lower part, when the top is cut off by a plane parallel to the base.

RULE. Add together the perimeters of the two ends, and multiply their sum by the slant height, taking half the product for the answer—As is evident, because the sides of the solid are trapezoids, having the opposite sides parallel.

PROBLEM IV.

To find the solid content of any Prism or Cylinder.

Find the area of the base, or end, whatever the figure of it may be, and multiply it by the length of the Prism or Cylinder, for the solid content.

PROBLEM V.

To find the content of any Pyramid or Cone.—See Fig. 16 and 17.

RULE. Find the area of the base, and multiply that area by the perpendicular height; then take one-third of the product for the content.

PROBLEM VI.

To find the solidity of the Frustum of a Cone or Pyramid.

RULE. Add into one sum the areas of the two ends, and the mean proportional between them; and take one-third of that sum for a mean area; which being multiplied by the perpendicular height or length of the frustum, will give its content.

PROBLEM VII.

To find the surface of a Sphere or any Segment.

See Fig. 18.

RULE I. Multiply the circumference of the sphere by its diameter, and the product will be the whole surface of it.

RULE 2. Square the diameter, and multiply that square by 3.1416, for the surface.

RULE 3. Square the circumference; then either multiply that square by the decimal .3183, or divide it by 3.1416, for the surface.

Note. For the surface of a Segment or Frustum, multiply the whole circumference of the Sphere by the height of the part required.

PROBLEM VIII.

To find the solidity of a Sphere or Globe.

RULE 1. Multiply the surface by the diameter, and take 1-6th of the product for the content;

or, which is the same thing, multiply the square of the diameter by the circumference, and take 1-6th of the product.

RULE 2. Take the cube of the diameter, and multiply it by the decimal .5236, for the content.

RULE 3. Cube the circumference, and multiply it by .01688, for the content.

PROBLEM IX.

To find the solid content of a Spherical Segment.

See Fig. 18.

RULE 1. From 3 times the diameter of the sphere, take double the height of the segment; then multiply the remainder by the square of the height, and the product by the decimal .5236, for the content.

RULE 2. To 3 times the square of the radius of the segment's base, add the square of its height; then multiply the sum by the height, and the product by .5236, for the content.

The foregoing Rules in Mensuration, both superficial and solid, are so simple, that it would be superfluous to give questions for their solutions; however, those that wish further information concerning them, will find them at large in *Hutton's Mathematics*, vol. 2, p. 26—52, from which the foregoing are taken.

SPECIFIC GRAVITY.

The specific gravity of a body is the proportional weight between that body and another of a known density; and water is admirably adapted to be the standard, as a solid foot of it weighs 1000 ounces avoirdupois.

TO FIND THE SPECIFIC GRAVITY OF A BODY.

PROBLEM I.

When the body is heavier than water.

RULE. Weigh it both in and out of water, and take the difference, which will be the weight lost in water; then say,

As the weight lost in water,
Is to the whole or absolute weight;
So is the specific gravity of water,
To the specific gravity of the body.

PROBLEM II.

When the body is lighter than water.

RULE. Annex to it a piece of another body heavier than water, so that the mass compounded of

the two may sink together. Weigh the denser body and the compound mass separately, both in and out of water; then find how much each loses in water, by subtracting its weight in water from its weight in air, and subtract the less of these remainders from the greater; then say,

As the last remainder,
Is to the weight of the light body in air;
So is the specific gravity of water,
To the specific gravity of the body.

PROBLEM III.

For a Fluid of any sort.

RULE. Take a piece of a body of known specific gravity, weigh it both in and out of the fluid, finding the loss of weight by taking the difference of the two; then say,

As the whole or absolute weight,
Is to the loss of weight;
So is the specific gravity of the solid,
To the specific gravity of the fluid.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

RULE. Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the compound and each ingredient; and

multiply each specific gravity by the difference of the other two; then say,

As the greatest product,
Is to the whole weight of the compound;
So is each of the other two products,
To the weights of the two ingredients.

EXAMPLE.

A composition of 112 lbs. being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and copper 9000.

8784, Composition,

9000, Copper,

7320, Tin.

$$9000 - 7320 = 680 \times 8784 = 5968320$$

$$8784 - 7320 = 1464 \times 9000 = 13176000$$

$$9000 - 8784 = 216 \times 7320 = 1581120$$

As 5968320 : 112 :: 13176000 : 100 = Copper } Weight of
112 - 100 = 12 = Tin } Ingredients

A Table of Specific Gravities of Bodies.

Platina (pure)	23000
Fine Gold	19400
Standard Gold	17724
Quicksilver (pure)	14000
Quicksilver (common)	13600
Lead	11325
Fine Silver	11091
Standard Silver	10535
Copper	9000
Copper Halfpence	8915
Gun Metal	8784
Cast Brass	8000
Steel	7850
Iron	7645
Cast Iron	7425
Coal	1250
Boxwood	1030
Sea Water	1030
Common Water	1000
Oak	925
Gunpowder close shaken	937
Do. in a loose heap	836
Tin	7320
Clear Crystal Glass,	3150
Granite	3000
Marble and Hard Stone	2700
Common Green Glass	2600
Flint	2570

Table continued.

Common Stone	2520
Clay	2160
Brick	2000
Common Earth	1984
Nitre	1900
Ivory	1825
Brimstone	1810
Solid Gunpowder	1745
Sand	1520
Ash	800
Maple	755
Elm	600
Fir	550
Charcoal	400
Cork	240
Air at a mean state	$1\frac{2}{9}$

Note. The several sorts of wood are supposed to be dry. Also, as a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in this Table express, not only the specific gravities of the several bodies, but also the weight of a cubic foot of each in avoirdupois ounces; and therefore, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known. Also, 100 cubic inches of common air weigh nearly $31\frac{1}{2}$ grains troy, or $1\frac{1}{7}$ drams avoirdupois. HUTTON.

TABLES OF THE WEIGHT OF MALLEABLE AND CAST IRON PLATES, BARS, &c.

TABLE of the Weight of a Square Foot of Cast and Malleable Iron, Copper and Lead, from 1-16th, to 1 Inch thick.

Thick.	Cast Iron.		Mall. Iron.		Copper.		Lead.	
	Libs.	Oz.	Libs.	Oz.	Libs.	Oz.	Libs.	Oz.
1 Sixteenth	2	6.6	2	7.8	2	15	3	11
2 —	4	13.3	4	15.6	5	14	7	6
3 —	7	4.	7	7.4	8	13	11	1
4 —	9	10.6	9	15.2	11	12	14	12
5 —	12	1.3	12	7.1	14	11	18	7
6 —	14	8.	14	14.9	17	10	22	2
7 —	16	14.7	17	6.7	20	9	25	13
8 —	19	5.3	19	14.5	23	8	29	8
9 —	21	12.	22	6.3	26	7	33	3
10 —	24	2.7	24	14.2	29	6	36	14
11 —	26	9.3	27	6.	32	5	40	9
12 —	29	-	29	13.8	35	4	44	4
13 —	31	6.7	32	5.6	38	3	47	15
14 —	33	13.4	34	13.4	41	2	51	10
15 —	36	4.	37	5.3	44	1	55	5
1 Inch	38	10.7	39	13.1	47	-	59	-

D

38. WEIGHT OF CAST AND MALL. IRON, &c.

TABLE of the Weight of a Lineal Foot of Malleable and Cast Iron Bars, from 6-16ths to 3 Inches square.

Sixteenths on the side.	Area in Square Sixteenths.	MALL IRON.	CAST IRON.	ROUND RODS.
		Ounces Weight.	Ounces Weight.	The 16ths on the side is the diameter of Rod. Ounces Weight.
6	36	7.4736	5.83
7	49	10.1724	7.99
8	64	13.2864	12.8960	10.43
9	81	16.8156	13.20
10	100	20.7600	16.30
11	121	25.1196	19.72
12	144	29.8944	29.0160	23.47
13	169	35.0844	27.53
14	196	40.6896	31.94
15	225	46.7100	36.44
1 Inch	256	53.1456	51.5840	41.50
1	289	59.9964	46.80
2	324	67.2624	52.47
3	361	74.9436	58.46
4	400	83.0400	80.6000	64.81
5	441	91.5516	71.41
6	484	100.4784	78.37
7	529	109.8204	85.66
8	576	119.5774	116.0640	93.27
9	625	129.7500	101.21
10	676	140.3376	109.46
11	729	151.3404	118.05
12	784	162.7584	157.9760	126.95
13	841	174.5916	136.19
14	900	186.8400	145.74
15	961	199.5036	155.62
2 Inches	1024	212.5824	206.3360	165.82
1	1089	226.0764	176.34
2	1156	239.9856	187.19
3	1225	254.3100	198.36
4	1296	269.0496	261.1440	209.86
5	1369	284.2044	221.68
6	1444	299.7744	233.83
7	1521	315.7596	246.30
8	1600	332.1600	322.4000	259.09
9	1681	348.9756	272.20
10	1764	366.2064	285.64
11	1849	383.8524	299.41
12	1936	401.9136	390.1040	313.49
13	2025	420.3900	327.91
14	2116	439.2816	342.64
15	2209	458.5884	357.70
3 Inches	2304	478.3104	464.2560	373.09

The foregoing Tables have been calculated from Hutton's Specific Gravities: those of Cast and Malleable Iron and Lead agree very nearly with those given by other authors; but the specific gravity of Copper, though heavier than that given by Hatchett, which is 8.800; still, from Copper being frequently alloyed with Lead, it is supposed that Hutton's, which is 9000, will be nearest the weight of Copper commonly used.



FALLING BODIES.

The motion described by Bodies freely descending by their own gravity, is, viz.—The Velocities are as the Times, and the Spaces as the Squares of the Times.—Therefore if the Times be as the numbers 1 2 3 4 &c.
 The Velocities will be also as . . 1 2 3 4 &c.
 The Spaces as their Squares . . 1 4 9 16 &c.
 and the Spaces for each time, as . 1 3 5 7 &c.
 namely, as the series of the odd numbers, which are the differences of the squares, denoting the whole spaces:—So that if the first series of numbers be seconds of time: *i. e.* . . 1" 2" 3" &c.
 Velocities in feet will be . . 32½ 64½ 96½ &c.
 Spaces in the whole times will be 16½ 64½ 144½ &c.
 Spaces for each second will be 16½ 48½ 80½ &c.

HUTTON.

FALLING BODIES.

The following TABLE shows the Spaces fallen through, and the Velocities acquired, at the end of each of 30 Seconds.

Time in Seconds.	SPACE.			VELOCITY.		
	Each Time.	As the Squares of the Time.	Fallen through in Feet & Inches.	As the Times.	Acquired in Feet & Inches.	
1	1	1	16 1	1	32	2
2	3	4	64 4	2	64	4
3	5	9	144 9	3	96	6
4	7	16	257 4	4	128	8
5	9	25	402 1	5	160	10
6	11	36	579 0	6	193	0
7	13	49	788 1	7	225	2
8	15	64	1029 4	8	257	4
9	17	81	1302 9	9	289	6
10	19	100	1608 4	10	321	8
11	21	121	1946 1	11	353	10
12	23	144	2316 0	12	386	0
13	25	169	2718 1	13	418	2
14	27	196	3152 4	14	450	4
15	29	225	3618 9	15	482	6
16	31	256	4117 4	16	514	8
17	33	289	4648 1	17	546	10
18	35	324	5211 0	18	579	0
19	37	361	5806 1	19	611	2
20	39	400	6433 4	20	643	4
21	41	441	7092 9	21	675	6
22	43	484	7784 4	22	707	8
23	45	529	8508 1	23	739	10
24	47	576	9264 0	24	772	0
25	49	625	10052 1	25	804	2
26	51	676	10872 4	26	836	4
27	53	729	11724 9	27	868	6
28	55	784	12609 4	28	900	8
29	57	841	13526 1	29	932	10
30	59	900	14475 0	30	965	0

EXAMPLE I.

To find the space descended by a body in 7" and the velocity acquired.

$$16.1 \times 49 = 788 \quad 1 \text{ feet of space.}$$

$$32.2 \times 7'' = 225 \quad 2 \text{ feet of velocity.}$$

Look into the Table at 7" and you have the answers.

EXAMPLE II.

To find the time of generating a velocity of 100 feet per second, and the whole space descended.

$$\frac{100 \times 12}{32.2 \times 12} = 3'' \frac{21}{193} \text{ Time.}$$

$$\frac{3'' \frac{21}{193} \times 100}{2} = 155 \frac{85}{193} \text{ Space descended.}$$

EXAMPLE III.

To find the time of descending 400 feet, and the velocity at the end of that time.

$$\sqrt{\frac{400 \times 12}{16.1 \times 12}} = 4'' \text{ 987 Time.}$$

$$\frac{400 \times 2}{4'' \text{ 987}} = 169.662 \text{ Velocity.}$$

Or these answers can be found from the Table by Proportion.

PENDULUM.

The vibrations of Pendulums are as the square roots of their lengths; and as it has been found by many accurate experiments, that the pendulum vibrating seconds in the latitude of London, is $39\frac{1}{8}$ inches long nearly, the length of any other pendulum may be found by the following Rule, viz. As the number of vibrations given, is to 60, so is the square root of the length of the pendulum that vibrates seconds, to the square root of the length of the pendulum that will oscillate the given number of vibrations:— or, As the square root of the length of the pendulum given, is to the square root of the length of the pendulum that vibrates seconds, so is 60 to the number of vibrations of the given pendulum.

Since the pendulum that vibrates seconds, or 60, is $39\frac{1}{8}$ inches long, the calculation is rendered simple; for $\sqrt{39\frac{1}{8}} \times 60 = 375$, a constant number, therefore 375, divided by the square root of the pendulum's length, gives the vibrations per minute, and divided by the vibrations per minute, gives the square root of the length of the pendulum.

EXAMPLE I.

How many vibrations will a pendulum of 49 inches long make in a minute?

$$\frac{375}{\sqrt{49}} = 53 \frac{1}{2} \text{ vibrations in a minute.}$$

EXAMPLE II.

What length of a pendulum will it require to make 90 vibrations in a minute?

$$\frac{375}{90} = 4.16 \sqrt[5]{4.16^3} = 17.3056 \text{ inches long.}$$

EXAMPLE III.

What is the length of a pendulum, whose vibrations will be the same number as the inches in its length?

$$\sqrt[5]{375^2} = 52 \text{ inches long; and 52 vibrations.}^*$$

* For $\sqrt{x} : \sqrt{39 \frac{1}{8}} :: 60 : x$ or $\sqrt{x} \times x = 375$

$$\sqrt{x} = \frac{375}{x} \Rightarrow x = \frac{375^2}{x^2}$$

$$\text{or } x \times x^2 = 375^2 = x^3 = 375^2$$

$$x = \sqrt[3]{375^2} = 52.002$$

MECHANICAL POWERS, &c.

THE Science of Mechanics is simply the application of **Weight and Power, or Force and Resistance**. The weight is the resistance to be overcome; the power is the force requisite to overcome that resistance. When the force is equal to the resistance, they are in a state of equilibrium, and no motion can take place; but when the force becomes greater than the resistance, they are not in a state of equilibrium, and motion takes place; consequently, the greater the force is to the resistance, the greater is the motion or velocity.

The Science of Equilibrium is called **STATICS**; the Science of Motion is called **DYNAMICS**.

Mechanical Powers are the most simple of mechanical applications to increase force, and overcome resistance. They are usually accounted six in number, viz. **The LEVER,—The WHEEL and AXLE,—The PULLEY,—The INCLINED PLANE,—The WEDGE,—and the SCREW.**

LEVER.

To make the principle easily understood, we must suppose the Lever an inflexible rod without weight; when this is done, the rule to find the equilibrium between the power and the weight, is,—Multiply the weight by its distance from the fulcrum, prop, or centre of motion, and the power by its distance from the same point: if the products are equal, the weight and power are in equilibrio; if not, they are to each other as their products.

EXAMPLE I.

A weight of 100 lbs on one end of a lever, is 6 inches from the prop, and a weight of 20 lbs at the other end, is 25 inches from the prop—What additional weight must be added to the 20 lbs, to make it balance the 100 lbs?

$$\frac{100 \times 6}{25} = 24 - 20 = 4 \text{ lbs weight to be added.}$$

EXAMPLE II.

A Block of 960 lbs is to be lifted by a lever 30 feet long, and the power to be applied is 60 lbs—on what part of the lever must the fulcrum be placed?

$\frac{960}{60} = 16$. that is, the weight is to the power as 16 is to 1; therefore the whole length $\frac{30}{16+1} = 1 \frac{5}{17}$, the distance from the block, and $30 - 1 \frac{5}{17} = 28 \frac{4}{17}$, the distance from the power.

EXAMPLE III.

A Beam 32 feet long, and supported at both ends, bears a weight of 6 tons, 12 feet from one end,—
What proportion of weight does each of the supports bear?

$\frac{12 \times 6}{32} = 2\frac{1}{4}$ tons, support at end farthest from
the weight.

$\frac{20 \times 6}{32} = 3\frac{3}{4}$ tons, support at end nearest the weight.

EXAMPLE IV.

A Beam supported at both ends, and 16 feet long, carries a weight of 6 tons, 3 feet from one end, and another weight of 4 tons, 2 feet from the other end: What proportion of weight does each of the supports bear?

$\frac{3 \times 6}{16} + \frac{14 \times 4}{16} = \frac{74}{16} = 4\frac{11}{16}$ tons, end at the 4 tons.

$\frac{2 \times 4}{16} + \frac{13 \times 6}{16} = \frac{86}{16} = 5\frac{6}{16}$ tons, end at the 6 tons.

When the weight of the lever is taken to account,
see Centre of Gravity.

WHEEL AND AXLE.

See Fig. 1. Plate 2.

The power gained by the Wheel and Axle, Wheel and Pinion, or Crane, is the effect of a double lever; for suppose the end of the lever ab is the radius of the rope barrel, and the radius of the wheel bc : again, the radius of the pinion working into the wheel, is de , and the length of the handle or winch is ef .

If the distance between ab is only one-third of the distance between bc , it is evident, that the point at c will go through three times the space to that of the point at a , when the lever revolves round its fulcrum b : the points d & f , in the other lever, are in the same proportion. The short end d acts upon the long end c : and if the end f goes through 9 inches, the end d will go through 3 inches, also the end c . If the end c goes through 3 inches, the end a will go through only 1 inch; therefore the power is to the weight as 9 is to 1; that is, If 9 lbs be hung at the end of the arm a , and 1 lib hung at the end of the arm f , they will balance each other. From this it is evident, that if you gain power, you lose speed; and by gaining speed, you lose power: hence the Rule is deduced—Multiply the power applied by its velocity, and the weight to be raised by its velocity.

EXAMPLE I.

A Weight of 94 tons is to be raised 360 feet in 15 minutes, by a power, the velocity of which is 220 feet per minute:—What is the power required?

$$\frac{360}{15} = 24 \text{ feet per minute, velocity of weight.}$$

$$24 \times 94 = \frac{2256}{220} = 10.2545 \text{ tons power required.}$$

EXAMPLE II.

A Stone weighing 986 lbs, is required to be lifted: What power must be applied, when the power is to the weight as 9 is to 2?

$$\frac{986 \times 2}{9} = \frac{1972}{9} = 219 \frac{2}{3} \text{ tons power.}$$

EXAMPLE III.

A Power of 18 lbs is applied to the winch of a crane, the length of which is 8 inches; the pulley makes 12 revolutions for 1 of the wheel, and the barrel is 6 inches diameter.

$$\frac{8 \times 2 \times 22}{7} = 50.28 \text{ circumference of the winch's circle.}$$

50.28 × 12 = 603.36 inches velocity of power on winch to 1 revolution of the barrel.

$$\frac{603.36 \times 18}{6 \times 22} = \frac{10860.48}{132} = 82.314 \text{ lbs weight,}$$

that can be raised by a power of 18 lbs on this crane.

PULLEY.

There are two kinds of Pulleys, the *fixed* and the *moveable*. From the fixed pulley no power is derived; it is as a common beam used in weighing goods, having the two ends of equal weight, and at the same distance from the centre of motion: the only advantage gained by the fixed pulley, is in changing the direction of the power.

From the moveable pulley power is gained; it operates as a lever of the second order; for if one end of a string be fixed to an immoveable stud, and the other end to a moveable power, the string doubled and the ends parallel, the pulley that hangs between is a lever; the fixed end of the string being the fulcrum, and the other the moveable end of the lever; hence the power is double the distance from the fulcrum, than is the weight hung at the pulley; and therefore the power is to the weight as 2 is to 1. This is all the advantage gained by one moveable pulley; for two, twice the advantage; for three, thrice the advantage; and so on for every additional moveable pulley.

From this the following Rule is derived:—Divide the weight to be raised by twice the number of moveable pulleys, and the quotient is the power required to raise the weight.

E

EXAMPLE I.

What power is requisite to lift 100 lbs, when two blocks of three pulleys, or sheives each, are applied, the one block moveable and the other fixed?

$$\frac{100}{6} = 16\frac{2}{3} \text{ lbs, the power required,}$$

$$3 \text{ sheives} \times 2 = 6.$$

EXAMPLE II.

What weight will a power of 80 lbs lift, when applied to a 4 and 5 sheived block and tackle, the 4 sheived block being moveable? *

$$80 \times 8 = 640 \text{ lbs weight raised.}$$

 INCLINED PLANE.

When a body is drawn up a vertical plane, the whole weight of the body is sustained by the power that draws or lifts it up: hence the power is equal to the weight.

* See Fig. 2. Plate 2.—If the strings be not parallel, but in the directions DA, DB, then the power A requisite to lift the weight C is as DE is to DC, and the strain upon the fixed point B is as CE is to CD.—HUTTON.

From this simple definition, it is easy to find the proportion between the power and the weight, when the strings are at any angle.

When a body is drawn along an horizontal (truly level) plane, it takes no power to draw it (save the friction occasioned by the rubbing along the plane.)

From these two hypotheses, if a body is drawn up an inclined plane, the power required to raise it is as the inclination of the plane; and hence when the power acts parallel to the plane, the length of the plane is to the weight, as the height of the plane is to the power; for the greater the angle, the greater the height.

EXAMPLE I.

What power is requisite to move a weight of 100 lbs up an inclined plane, 6 feet long and 4 feet high?

If $6 : 4 :: 100 : 66\frac{2}{3}$ lbs power.

EXAMPLE II.

A power of 68 lbs, at the rate of 200 feet per minute, is applied to pull a weight up an inclined plane, at the rate of 50 feet per minute—When the plane is 37 feet long and 12 feet high, how much will be the weight drawn? *

As $12 : 37 :: 68 \times 200 : 50 \times 838\frac{4}{5}$

$$\frac{68 \times 200 \times 37}{12 \times 50} = \frac{503200}{600} = 838\frac{4}{5} \text{ lbs weight}$$

* See Fig. 3. Plate 2. When the direction of the power is at any angle than parallel to the plane.

Draw BC at right angles to the direction of the power P, then the Weight is to the Power as AD is to DC—that is, $W : P :: AD : DC$

WEDGE.

The Wedge is a double inclined plane, and therefore subject to the same Rules, or the following Rule, which is particularly for the Wedge; but drawn from its near connection to the inclined plane, is,—If the power acts perpendicularly upon the head of the wedge, the power is to the pressure which it exerts perpendicularly on each side of the wedge, as the head of the wedge is to its side: hence, it is evident, that the sharper, or thinner the wedge is, the greater will be the power.

But the power of the Wedge being not directly according to its length and thickness, but to the length and width of the split, or rift, in the wood to be cleft, the rule therefore is of little use in practice; besides, the wedge is very seldom used as a power; for these reasons, the nature of its properties and effects need not be here discussed.

SCREW.

The Screw is a cord wound in a spiral direction round the periphery of a cylinder, and is therefore an inclined plane, the length being the circumference of the cylinder, and the height, the distance between two consecutive cords, or threads of the

Screw, hence, the Rule is derived:—As the circumference of the Screw is to the Pitch, or distance between the threads; so is the Weight to the Power.

When the Screw turns, the cord or thread runs in a continued ascending line round the centre of the cylinder, and the greater the radius of the cylinder, the greater will be the length of the plane to its height, consequently the greater the power.—A lever fixed to the end of the screw will act as one of the second order, and the power gained will be as its length, to the radius of the cylinder; or the circumference of the circle described by it, to the circumference of the cylinder; hence, an addition to the Rule is produced, which is,—If a lever is used, the circumference of the lever is taken for, or instead of, the circumference of the screw.

EXAMPLE I.

What is the power requisite to raise a weight of 8000 lbs by a Screw of 12 inches circumference and 1 inch pitch?

As $12 : 1 :: 8000 : 666\frac{8}{12}$ lbs = power at the circumference of the screw.

EXAMPLE II.

How much would be the power if a lever of 30 inches was applied to the screw?

Circumference of 30 inches = 1884
 As $1884 : 1 :: 8000 : 42\frac{560}{1884}$ lbs = power with a lever of 30 inches long

STRENGTH OF MATERIALS.

AFTER considering the Mechanical Powers, which are the analytic parts of all Machines, the next step is to consider the Strength of the Materials of which Machinery is composed—this knowledge being of the greatest importance to the Mechanic, by enabling him to adjust, with respect to magnitude, the various parts of the machine, that the strength of each part may be proportional to the stress it has to sustain.

COHESIVE STRENGTH OF BEAMS, BARS, &c.

The cohesive strength of a body, is that force by which its fibres or particles resist separation, therefore the more particles that are in a body, the greater will be the power requisite to tear them

asunder, or according to the Rule, that the strength of bodies are as the area of their cross sections.

The knowledge in this property of bodies is very limited, there being very few experiments made on which to build a data, and these few do not agree.

The following are the results of experiments made by Mr. Emerson, which state the load that may be safely borne by a square inch rod of each.

	Pounds Avoirdupois.
Iron Rod, an inch square, will bear	76,400
Brass	35,600
Hempen Rope	19,600
Ivory	15,700
Oak, Box, Yew, Plum-tree, . .	7,850
Elm, Ash, Beech	6,070
Walnut, Plum	5,360
Red Fir, Holly, Elder, Plane, Crab	5,000
Cherry, Hazel.	4,760
Alder, Asp, Birch, Willow . . .	4,290
Lead	430
Free Stone	914

Mr. Barlow's opinion of this table is, "We shall only observe here, that they all fall very short of the ultimate strength of the woods to which they refer.—*See Barlow's Essay on the strength of Timber. Art. 3.*

Mr. Emerson also gives the following practical Rule, viz. "That a Cylinder, whose diameter is *d* inches, loaded to one-fourth of its absolute strength, will carry as follows:

	<i>Cwt.</i>
Iron,	135 × d^2
Good Rope, . .	22 × d^2
Oak,	14 × d^2
Fir,	9 × d^2

Captain S. Brown made an experiment on Welsh Pig Iron, and the result is described as follows:

“A Bar of Cast Iron, Welsh Pig, $1\frac{1}{4}$ inch square, 3 feet 6 inches long, required a strain of 11 tons, 7 cwt. (25,424 lbs,) to tear it asunder, broke exactly transverse; without being reduced in any part; quite cold when broken; particles fine; dark bluish grey colour.”—From this experiment, it appears that 16,265 lbs. will tear asunder a square inch of Cast Iron.

Mr. G. Rennie also made some experiments on Cast Iron, and the result was, “that a Bar one inch square, cast horizontal, will support a weight of 18,656 lbs—and one cast vertical, will support a weight of 19,488 lbs.

There have been several experiments made on Malleable Iron, of various qualities, by different Engineers.

The mean of Mr. Telford's experiments, is $29\frac{1}{4}$ tons.
 The mean of Capt. S. Brown's do. is 25 do.
 and the mean between these two means, is 27 tons, nearly; which may be assumed as the medium strength of a Malleable Iron Bar 1 inch square.
See Barlow's Essay, page 235.

From a mean, derived by experiments, performed by Mr. Barlow, it appears that the strength of direct cohesion, on a square inch of

	<i>Libs.</i>
Box . . . is about . . .	20,000
Ash . . . — . . .	17,000
Teak . . . — . . .	15,000
Fir . . . — . . .	12,000
Beech . . . — . . .	11,500
Oak . . . — . . .	10,000
Pear . . . — . . .	9,800
Mahogany — . . .	8,000

Each of these weights may be taken as a correct data for the cohesive strength of the wood to which they belong; but this is the absolute and ultimate strength of the fibres; and therefore, if the quantity that may be safely borne be required, not more than two-thirds of the above values must be used.

TRANSVERSE STRENGTH OF BEAMS, BARS, &c.

If a beam be supported at both ends, and loaded in the middle, it will bend; (which is called deflection); and if the load be increased, it will break, (which is called fracture).—If a beam 2 inches deep and one inch broad, supports a given weight, another beam of the same depth, and double the breadth, will support double the weight:—hence, beams of

the same depth are to each other as their breadths:—again, If a beam 2 inches deep, and 1 inch broad, support a given weight, another beam of 4 inches deep, and 1 inch broad, will support four times the weight;—hence, beams of equal breadths are to each other as the squares of their depths:—again, If a beam of a given cross section 1 foot long, support a known weight, another beam of the same cross section but 2 feet long, will support only half the known weight;—hence, beams of equal dimensions are to each other inversely as their lengths; therefore, the strength of beams is directly as their breadths and square of their depths, and inversely as their lengths; and if cylindrical, as the cubes of their diameters.

PRACTICAL PROBLEMS *for the Transverse Strength of Timber.**

TABLE OF MULTIPLICANDS.

English Oak	1426
Canadian do.	1766
Ash	2026
Beech	1556
Elm	1013
Pitch Pine	1632
Red Pine	1341
Fir	1100
Larch	1127

* See Barlow's Essay on the strength and stress of Timber, *Art.* 149.

PROBLEM I.

To find the ultimate transverse strength of any Rectangular Beam of Timber, fixed at one end, and loaded at the other.

RULE. Multiply the number in the Table of Multiplicands, by the breadth and square of the depth, both in inches, and divide that product by the length also in inches; the quotient will be the weight in lbs.*

EXAMPLE I.

What weight will it require to break a beam of Fir, the breadth being 2 inches, depth 6 inches, and length 20 feet?

$$\frac{1100 \times 36 \times 2}{240} = 330 \text{ lbs.}$$

EXAMPLE II.

What is the weight requisite to break a beam of Ash, 7 inches square, 3 feet from the wall?

$$\frac{2026 \times 7^3}{36} = 19303\frac{10}{9} \text{ lbs.}$$

* When the beam is loaded uniformly throughout its length, the same rule will still apply, only the result must be doubled.

PROBLEM II.

To compute the ultimate transverse strength of any Rectangular Beam, when supported at both ends and loaded in the centre.

RULE. Multiply the number in the Table of Multiplicands, by the square of the depth in inches, and four times the breadth; divide that product by the length in inches, and the quotient will be the weight.

EXAMPLE I.

What weight will break a beam of English Oak 7 inches broad, 9 inches deep, and 30 feet between the props?

$$\frac{1426 \times 81 \times 28}{360} = 8983 \frac{488}{560} \text{ lbs.}$$

EXAMPLE II.

A beam of beech, 7 inches deep, 4 inches broad, and 10 feet long, supports a weight of 4 tons, what additional weight will require to be added to break the beam?

$$\frac{1556 \times 49 \times 16}{120} = 10332 - 8960 = 1372 \text{ lbs.}$$

When the beam is uniformly loaded throughout its length, the result must be doubled, *i. e.* it will support double the weight.

When the Beam is fixed at both ends and loaded in the middle, one-half of the result must be added; and if the weight is laid uniformly along its length, the result must be tripled.

These Problems are taken from Barlow's Essay, as before quoted: he, however, gives a second Rule to each of the Problems, in which the angle of deflection is considered. These Rules give higher results than those here stated; but for practice the first Rule is the best, being more simple and safe.

It is considered that two-thirds of the result is sufficient to lay upon a beam for a permanent load.

PRACTICAL PROBLEMS *for the Transverse strength of Cast Iron Beams.**

PROBLEM I.

To find the breadth of a uniform Cast Iron Beam to bear a given weight in the middle.

RULE 1. Multiply the length of bearing in feet, or the length between the supports, by the weight to be supported in libs; and divide this product by 850 times the square of the depth in inches; the quotient will be the breadth in inches required.

* See Tredgold's *Practical Essay on the Strength of Cast Iron*, p. 80.

RULE 2. Multiply the length of bearing in feet, by the weight to be supported in lbs, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the n^{th} part of the depth, n representing any number at will. Then the Rule is as follows:—

Multiply n times the length in feet, by the weight in lbs; divide this product by 850, and the cube root of the quotient will be the depth required; and the breadth will be the n^{th} part of the depth.

Note. It may be remarked here, that the Rules are the same for inclined as for horizontal beams, when the horizontal distance between the supports is taken for the length of bearing.

EXAMPLE I.

What is the breadth of a Beam 20 feet long, 15 inches deep, and to be loaded with 13 tons?

$$13 \text{ tons} = 29120 \text{ lbs.}$$

$$\frac{29120 \times 20}{15^2 \times 850} = 3.045 \text{ inches broad.}$$

EXAMPLE II.

What is the depth of a Beam 20 feet long, 3 inches broad, and to support a weight of 13 tons?

$\frac{20120 \times 20}{850 \times 3} = 225$, the square root of which is =
15 inches, the depth required.

EXAMPLE III.

What are the cross sectional dimensions of a Beam 30 feet long, and of sufficient strength to support a weight of 10 tons; the depth being twice the breadth?— n will therefore be = 2

10 tons = 22400 lbs. Length = 30. $30 \times 2^n = 60$
 $\frac{22400 \times 60}{850} = 1581$, the cube root of which is nearly

$11\frac{1}{2}$, which is equal to the depth in inches: the breadth is the half of the depth = $5\frac{1}{4}$ inches.

PROBLEM II.

To find the breadth, when the load is not in the middle between the supports.

RULE. Multiply the short length by the long length, and four times this product divided by the whole length between the supports, will give the effective leverage of the load in feet; this quotient being used instead of the length, in any of the Rules in the foregoing Problem, the breadth and depth will be found by them.

EXAMPLE.

What are the cross sectional dimensions of a Beam 12 feet long, supporting a weight of 15 tons,

3 feet from the one end, when the breadth is a fourth of the depth?

$$\frac{3 \times 9 \times 4}{12} = 9 \quad 9 \times 4 = 36 \quad 15 \text{ tons} = 33600 \text{ lbs.}$$

$\frac{33600 \times 36}{850} = 1423$, the cube root of which is $= 11\frac{1}{4}$, the depth: the breadth will be $\frac{11\frac{1}{4}}{4} = 2\frac{5}{16}$

PROBLEM III.

To find the breadth when the load is uniformly distributed over the length of the beam.

RULE. The same Rules apply as in Prob. 1, only the divisor is changed from 850 to 1700, *i. e.* when the load is uniformly distributed over the length of the beam, it supports double the weight than when the load is laid on the middle.

Note. Examples in Problem 1. apply to this Problem, only changing the divisors, or halving the quotients.

PROBLEM IV.

To find the dimensions, when a beam is fixed at one end and loaded at the other, or when it is supported at the middle and loaded at both ends.

RULE. Take the horizontal length of the projection of the beam, when fixed at one end, for the length, and apply the Rules in Prob. 1. only using the divisor 212 instead of 850.

When the beam is supported any where between the two ends, multiply the length from the prop by

the weight hung at the end, and apply the remainder of the Rule as in Prob. 1. only using 212 for 850.

When the load is uniformly distributed over the length of the projection, employ 425 instead of 212 as a divisor.

Note. The Rules of this Problem apply to the teeth of wheels, the length being the length of the teeth, and the depth, the thickness of the teeth.

Example to this Note.

Let the greatest power acting at the pitch line of the wheel be 6000 lbs, and the thickness of the teeth $1\frac{1}{2}$ inch, the length of the teeth being $\frac{1}{4}$ foot; What is the breadth of the teeth?

$$\frac{6000 \times .25}{212 \times 1.5^2} = \frac{1500}{477} = 3.14 \text{ inches the breadth;}$$

but to allow for wearing by friction, this quotient is doubled, or $6\frac{1}{2}$ inches = the breadth of the teeth, or face of the wheel

PROBLEM V.

To find the diameter of a solid cylinder to support a given weight in the middle—between the middle and the end,—and when the weight is uniformly distributed over the length—also when fixed at one end.

When the weight is placed in the middle.

RULE. Multiply the weight in lbs by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.

F 3

When the weight is between the middle and end.

RULE. Multiply the short end by the long end; then multiply that product by 4 times the weight in lbs. Divide this product by 500 times the length in feet, and the cube root of the quotient will be the diameter in inches.

When the load is uniformly distributed over the length.

RULE. Multiply the length in feet by the weight in lbs, and one-tenth of the cube root of the product will be the diameter in inches.

When fixed at one end, and the load applied at the other.

RULE. Multiply the length of projection in feet by the weight in lbs, and the 5th part of the cube root of this product will be the diameter in inches.

The Rules for the deflection of Beams and Bars are here omitted, being considered, that, in most of practical cases, the deflection is of little importance; however, when it is of importance, reference to Barlow's Essay on the strength of Timber, and Tredgold's Essay on the strength of Iron, will satisfy all inquiries. These books ought to be in the possession of every Mechanic, as they give the most comprehensive, and most correct data for the strength of materials, of any that have yet appeared.

STRENGTH OF THE JOURNALS OF SHAFTS.

LATERAL STRENGTH.

THE Rules in Problem 5. of last article, can be here applied. Mr. Robertson Buchanan, in his Essay on the Strength of Shafts, uses the following Rule, which is simple enough, and easy to be remembered; but the above-mentioned Rules are the most correct, and ought to be used on all occasions.

Mr. Buchanan's Rule is,—“The cube root of the weight in cwts. is nearly equal to the diameter of the Journal.”—“*Nearly equal*,—being prudent to make the Journal a little more than less, and to make a due allowance for wearing.”

EXAMPLE.

What is the diameter of the Journal of a Water Wheel Shaft, 13 feet long, the weight of the Wheel being 15 tons?

By Mr. B.'s Rule. $\sqrt[5]{15 \times 20} = 6.7$ or 7 inches diam^r.

By Mr. Tredgold's Rule.

Weight in the middle. $\left\{ \frac{38600 \times 13}{500} = 873 \right.$ $\sqrt[5]{873} = 9\frac{1}{2}$ inches diam^r.

Weight equally distributed. $\left\{ 38600 \times 13 = 496800 \right.$ $\frac{\sqrt[5]{496800}}{10} = 7.65$ In.

TO RESIST TORSION OR TWISTING.

It is obvious that the strength of revolving Shafts are directly as the cubes of their diameters and revolutions; and inversely, as the resistance they have to overcome.

Mr. Robertson Buchanan, in his Essay on the strength of Shafts gives the following data, deduced from several experiments, viz. That the Fly Wheel Shaft of a 50 horse power engine, at 50 revolutions per minute, requires to be $7\frac{1}{2}$ inches diameter, and therefore, the cube of this diameter, which is = 421.875, serves as a multiplier to all other shafts in the same proportion; and taking this as a standard, he gives the following Multipliers, viz.

For the Shaft of a Steam Engine, Water Wheel, or any Shaft connected with a first power, - - - - -	} 400
For Shafts in inside of Mills, to drive smaller Machinery, or connected with the Shafts above, - - - - -	
For the small Shafts of a Mill or Machinery,	100

From the foregoing the following Rule is derived, viz.

The number of horses' power a Shaft is equal to, is directly as the cube of the diameter and number of revolutions, and inversely as the above Multipliers.

Note. Shafts here are understood as the Journals of Shafts, the bodies of Shafts being generally made square.

EXAMPLE I.

When the Fly Wheel Shaft of a 45 horse power Steam Engine makes 90 revolutions per minute, what is the diameter of the Journal?

$$\frac{45 \times 400}{90} = 200 \quad \sqrt[3]{200} = 5\frac{8}{10} \text{ inches diameter.}$$

EXAMPLE II.

The velocity of a Shaft is 80 revolutions per minute, and its diameter is 3 inches: What is its power?

$$\frac{3^3 \times 80}{400} = 5.4 \text{ horse power.}$$

EXAMPLE III.

What will be the diameter of the Shaft in the first Example, when used as a Shaft of the second Multiplier?*

$$\frac{5.8}{1.25} = 4.64, \text{ or } \sqrt[3]{\frac{44 \times 200}{90}} = 4\frac{8}{10} \text{ inches diameter.}$$

The following is a Table of the diameters of Shafts, being the First Movers, or having 400 for their multipliers.

* The diameters of the second movers will be found by dividing the numbers in the Table by 1.25, and the diameters of the third movers, by dividing the numbers by 1.56.

STRENGTH OF THE

TABLE.

DIAMETERS OF THE JOURNALS OF FIRST MOVERS.

Horses' Power.	REVOLUTIONS.									
	10	15	20	25	30	35	40	45	50	55
4	5.5	4.8	4.5	4.	3.7	3.8	3.5	3.3	3.2	3.1
5	5.9	5.1	4.7	4.4	4.1	3.9	3.7	3.6	3.5	3.3
6	6.3	5.5	5.	4.6	4.4	4.1	4.	3.8	3.7	3.6
7	6.6	5.8	5.2	4.9	4.6	4.4	4.2	4.	3.9	3.7
8	6.9	6.	5.5	5.1	4.8	4.6	4.4	4.2	4.1	4.
9	7.2	6.3	5.7	5.5	5.	4.8	4.5	4.4	4.2	4.1
10	7.4	6.6	5.9	5.6	5.2	4.9	4.7	4.6	4.4	4.2
12	7.9	6.9	6.3	5.8	5.6	5.4	5.2	5.	4.8	4.6
14	8.3	7.2	6.7	6.2	5.9	5.6	5.4	5.2	5.	4.7
16	8.7	7.6	7.1	6.6	6.1	5.8	5.6	5.4	5.2	5.
18	9.	7.9	7.5	7.	6.6	6.2	5.8	5.6	5.4	5.2
20	9.3	8.1	7.4	7.2	6.6	6.4	5.9	5.7	5.6	5.4
25	10.	8.5	8.	7.4	7.1	6.8	6.3	6.	5.9	5.6
30	10.7	9.3	8.4	7.9	7.4	7.1	6.9	6.7	6.5	6.3
35	11.4	9.8	8.9	8.4	7.9	7.4	7.1	6.9	6.6	6.5
40	11.7	10.5	9.3	8.8	8.3	7.8	7.4	7.2	6.9	6.7
45	12.	10.6	9.7	9.2	8.7	8.1	7.6	7.4	7.	6.8
50	12.6	11.	10.	9.3	9.	8.5	8.	7.8	7.4	7.3
55	13.4	11.4	10.4	9.8	9.1	8.8	8.4	8.	7.5	7.4
60	13.6	12.	10.8	10.	9.3	9.	8.6	8.2	7.7	7.6

INCHES DIAMETER.

TABLE CONTINUED.

Horse Power.	REVOLUTIONS.									
	60	65	70	75	80	85	90	95	100	105
4	3.	2.9	2.9	2.8	2.7	2.7	2.6	2.6	2.6	2.5
5	3.3	3.2	3.1	3.	3.	2.9	2.9	2.8	2.8	2.7
6	3.5	3.5	3.4	3.3	3.2	3.2	3.	3.	2.9	2.9
7	3.6	3.6	3.5	3.4	3.4	3.3	3.3	3.2	3.1	3.1
8	3.9	3.8	3.7	3.6	3.5	3.5	3.4	3.4	3.3	3.2
9	4.	3.8	3.7	3.7	3.6	3.6	3.5	3.5	3.4	3.3
10	4.1	4.	3.9	3.8	3.7	3.7	3.6	3.6	3.5	3.4
12	4.4	4.3	4.2	4.1	4.	3.9	3.8	3.8	3.7	3.6
14	4.5	4.4	4.4	4.3	4.2	4.1	4.	4.	3.9	3.8
16	4.8	4.7	4.6	4.5	4.4	4.4	4.3	4.2	4.1	4.
18	5.	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.2
20	5.2	5.1	5.	4.8	4.6	4.6	4.5	4.5	4.4	4.4
25	5.5	5.4	5.3	5.2	5.1	4.9	4.8	4.7	4.6	4.6
30	5.9	5.8	5.7	5.6	5.5	5.3	5.2	5.1	5.	4.9
35	6.3	6.1	5.9	5.7	5.6	5.5	5.4	5.3	5.2	5.2
40	6.6	6.4	6.2	6.	5.9	5.8	5.7	5.6	5.6	5.5
45	6.7	6.5	6.4	6.2	6.1	6.	5.9	5.8	5.7	5.6
50	7.2	6.9	6.8	6.6	6.5	6.4	6.2	6.	5.9	5.8
55	7.3	7.2	7.	6.7	6.6	6.5	6.3	6.2	6.1	6.
60	7.4	7.3	7.2	6.9	6.8	6.8	6.7	6.6	6.4	6.2

INCHES DIAMETER.

It is a well known fact, that a cast iron Rod will sustain more torsional pressure, than a malleable iron Rod of the same dimensions.—That is, a malleable iron rod will be twisted by a less weight than what is required to wrench a cast iron rod of the same dimensions.

When the strength of malleable iron is less than that of cast iron to resist torsion; it is stronger than cast iron to resist lateral pressure, and that strength is in proportion as 9 is to 14.

From the foregoing, it is easy for the Mill-wright to make his shafts of the iron best suited to overcome the resistance to which they will be subject, and the proportion of the diameters of their Journals, according to the iron of which they are made:—for example; What will be the diameter of a malleable iron Journal, to sustain an equal weight with a cast iron Journal of 7 inches diameter?

$$7^3 = 343$$

14 : 343 :: 9 : $220\frac{1}{2}$ now $\sqrt[5]{220.5} = 6.04$ inches diameter.

STRENGTH OF WHEELS.

THE arms of Wheels are as levers fixed at one end and loaded at the other, and consequently the greatest strain is upon the end of the arm next the axle; for that reason all arms of wheels should be strongest at that part, and tapering towards the rim.

The Rule for the breadth and thickness of arms, according to their length and number in the wheel, is as follows. (*See Tredgold's Essay, page 114.*) Multiply the power or weight acting at the end of the arm by the cube of its length; the product of which, divided by 2656 times the number of arms multiplied by the deflection, will give the breadth and cube of the depth.

EXAMPLE.

Suppose the force acting at the circumference of a spur wheel to be 1600 lbs, the radius of wheel 6 feet, and number of arms 8, and let the deflection not exceed $\frac{1}{10}$ of an inch.

$$\frac{1600 \times 6^3}{2656 \times 8 \times .1} = 163 = \text{breadth and cube of the depth.}$$

Let the breadth be 2.5 inches, therefore $\frac{163}{2.5} = 65.2$, which is equal to the cube of the depth: now the

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cube root of 65.2 is nearly 4.03 inches; this, consequently, is the depth, or dimension, of each arm in the direction of the force.

Note. When the depth at the rim is intended to be half that at the axes, use 1640 as a divisor instead of 2656.

The teeth are as beams, or cantilevers, fixed at one end and loaded at the other, the Rule applying direct to them (*See Tredgold's Essay, Art. 121*) where the length of the beam is the length of the teeth, and the depth, the thickness of the teeth. For the better explanation of the Rule the following Example is given.

EXAMPLE.

The greatest power acting at the pitch line of the wheel is 6000 lbs, and the thickness of the teeth $1\frac{1}{2}$ inch, the length of the teeth being 0.25 feet; it is required to determine the breadth of the teeth?

$$\frac{6000 \times 0.25}{212 \times 1.5^2} = \frac{1500}{477} = 3.2 \text{ inches the breadth}$$

In order that the teeth may be capable of offering a sufficient resistance after being worn by friction, the breadth thus found should be doubled; therefore, in the above Example, the breadth should be 6.4, or say $6\frac{1}{2}$ inches.

Mr. Carmichael* gives the following data, gleaned from experiments, which is therefore valuable, and of much use to the practical mechanic.

* See Robertson's *Dictionary on the Teeth of Wheels.*

RULE. Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length; the quotient will be the proportional strength in horses' power, with a velocity of 2.27 feet per second.

EXAMPLE.

What is the power of a wheel, the teeth of which are 6 inches broad, 1.5 inch thick, and 1.8 inch long, and revolving at the velocity of 3 feet per second?

$$\frac{1.5^2 \times 6}{1.8} = \frac{13.5}{1.8} = 7.5 \text{ strength at 2.27 feet per sec.}$$

$$\text{then } 2.27 : 7.5 :: 3 = \frac{7.5 \times 3}{2.27} = 9.91 \text{ horse-power.}$$

RULE. The pitch is found by multiplying the thickness by 2.1, and the length is found by multiplying the thickness by 1.2.

EXAMPLE.

The thickness being 2 inches, what is the pitch and length?

$$2 \times 2.1 = 4.2 \text{ Pitch.}$$

$$2 \times 1.2 = 2.4 \text{ Length.}$$

Note. The breadth of the teeth, as commonly executed by the best Masters, seems to be from about twice to thrice the pitch.

TABLE.

Pitch in Inches.	Thick- ness in Inches.	Breadth in Inches.	Length in Inches.	Horses Power at 2.27 feet per Second.	H. P. at 3 feet per Second.	H. P. at 6 feet per Second.	H. P. at 11 feet per Second.
4.2	2.	8.	2.40	13.33	17.61	35.23	64.6
3.99	1.9	7.6	2.28	13.03	15.90	31.80	58.30
3.78	1.8	7.2	2.16	10.80	14.27	28.54	52.32
3.57	1.7	6.8	2.04	9.63	12.72	25.54	46.68
3.36	1.6	6.4	1.92	8.53	11.27	22.54	41.32
3.15	1.5	6.	1.80	7.50	9.91	19.82	36.33
2.94	1.4	5.6	1.68	6.53	8.63	17.26	31.64
2.73	1.3	5.2	1.56	5.63	7.44	14.88	27.28
2.52	1.2	4.8	1.44	4.80	6.34	12.68	23.24
2.31	1.1	4.4	1.32	4.03	5.32	10.64	19.54
2.10	1.	4.	1.20	3.33	4.40	8.81	16.15
1.89	.9	3.6	1.08	2.70	3.57	7.14	13.09
1.68	.8	3.2	.96	2.13	2.81	5.62	10.33
1.47	.7	2.8	.84	1.63	2.15	4.30	7.88
1.26	.6	2.4	.72	1.20	1.59	3.18	5.83
1.05	.5	2.	.60	.83	1.10	2.20	4.03

VELOCITY OF WHEELS.

WHEELS are for conveying motion to the different parts of a machine, at the same, or at a greater, or less velocity, as may be required.—When two wheels are in motion their teeth act on one another alternately, and consequently, if one of these wheels has 40 teeth, and the other 20 teeth, the one with 20 will turn twice upon its axis, for one revolution of the wheel with 40 teeth.—From this the Rule is taken, which is,—As the velocity required is to the number of teeth in the driver, so is the velocity of the driver to the number of teeth in the driven.

Note. To find the proportion that the velocities of the wheels in a train should bear to one another, subtract the less velocity from the greater, and divide the remainder by the number of one less than the wheels in the train; the quotient will be the number rising in Arithmetical progression, from the least to the greatest velocity of the train of wheels.

EXAMPLE I.

What is the number of teeth in each of three wheels to produce 17 revolutions per minute, the driver having 107 teeth, and making 3 revolutions per minute?

$17 - 3 = \frac{14}{3 - 1} = 7$, therefore 3 10 17 are the velocities of the three wheels.

By the Rule. $\left\{ \begin{array}{l} 10 : 107 :: 3 : 32 = \frac{107 \times 3}{10} = 32 \text{ teeth.} \\ 17 : 32 :: 10 : 19 = \frac{32 \times 10}{17} = 19 \text{ teeth.} \end{array} \right.$

EXAMPLE II.

What is the number of teeth in each of 7 wheels, to produce 1 revolution per minute, the driver having 25 teeth, and making 56 revolutions per minute?

$\frac{56 - 1}{7 - 1} = \frac{55}{6} = 9$, therefore 56 46 37 28 19 10 1, progressional velocities.

46 : 25 :: 56 : 30 Teeth.

37 : 30 :: 46 : 37 —

28 : 37 :: 37 : 49 —

19 : 49 :: 28 : 72 —

10 : 72 :: 19 : 137 —

1 : 137 :: 10 : 1370 —

It will be observed that the last wheel, in the foregoing Example, is of a size too great for application;

to obviate this difficulty, which frequently arises in this kind of training, wheels and pinions are used, which give a great command of velocity.—Suppose the velocities of last Example, and the train only of 2 wheels and 2 pinions.

$$\frac{56 - 1}{4 - 1} = \frac{55}{3} = 18, \text{ therefore, } 56 \ 19 \ 1, \text{ are the}$$
 progressional velocities.

$19 : 25 :: 56 : 74 =$ teeth in the wheel driven by the first driver, and $1 : 10 :: 19 : 190 =$ teeth, in the second driven wheel, 10 teeth being in the driving pinion.

25 drivers	74 driven.
10 ———	190 ———

The following is a Table of the Radii of Wheels, from ten to three hundred teeth, the pitch being 2 inches.

The radius for any other pitch may be found by the following analogy:—As two Inches is to the Radius in the Table, so is the new Pitch to the new Radius.

TABLE.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
10	3.236	46	14.654	82	26.108	118	37.565
11	3.549	47	14.972	83	26.426	119	37.883
12	3.864	48	15.290	84	26.741	120	38.202
13	4.179	49	15.608	85	27.063	121	38.520
14	4.494	50	15.926	86	27.381	122	38.838
15	4.810	51	16.244	87	27.699	123	39.156
16	5.126	52	16.562	88	28.017	124	39.475
17	5.442	53	16.880	89	28.336	125	39.793
18	5.759	54	17.198	90	28.654	126	40.111
19	6.076	55	17.517	91	28.972	127	40.429
20	6.392	56	17.835	92	29.290	128	40.748
21	6.710	57	18.153	93	29.608	129	41.066
22	7.027	58	18.471	94	29.927	130	41.384
23	7.344	59	18.789	95	30.245	131	41.703
24	7.661	60	19.107	96	30.563	132	42.021
25	7.979	61	19.425	97	30.881	133	42.339
26	8.296	62	19.744	98	31.200	134	42.657
27	8.614	63	20.062	99	31.518	135	42.976
28	8.931	64	20.380	100	31.836	136	43.294
29	9.249	65	20.698	101	32.155	137	43.612
30	9.567	66	21.016	102	32.473	138	43.931
31	9.885	67	21.335	103	32.791	139	44.249
32	10.202	68	21.653	104	33.109	140	44.567
33	10.520	69	21.971	105	33.427	141	44.885
34	10.838	70	22.289	106	33.746	142	45.204
35	11.156	71	22.607	107	34.064	143	45.522
36	11.474	72	22.926	108	34.382	144	45.840
37	11.792	73	23.244	109	34.700	145	46.158
38	12.110	74	23.562	110	35.018	146	46.477
39	12.428	75	23.880	111	35.337	147	46.795
40	12.746	76	24.198	112	35.655	148	47.113
41	13.064	77	24.517	113	35.974	149	47.432
42	13.382	78	24.835	114	36.292	150	47.750
43	13.700	79	25.153	115	36.611	151	48.068
44	14.018	80	25.471	116	36.929	152	48.387
45	14.336	81	25.790	117	37.247	153	48.705

TABLE CONTINUED.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
154	49.023	191	60.800	228	72.577	265	84.354
155	49.341	192	61.118	229	72.895	266	84.673
156	49.660	193	61.436	230	73.214	267	84.991
157	49.978	194	61.755	231	73.532	268	85.309
158	50.296	195	62.073	232	73.850	269	85.627
159	50.615	196	62.392	233	74.168	270	85.946
160	50.933	197	62.710	234	74.487	271	86.265
161	51.251	198	63.028	235	74.805	272	86.582
162	51.569	199	63.346	236	75.123	273	86.900
163	51.888	200	63.665	237	75.441	274	87.219
164	52.206	201	63.983	238	75.760	275	87.537
165	52.524	202	64.301	239	76.078	276	87.855
166	52.843	203	64.620	240	76.397	277	88.174
167	53.161	204	64.938	241	76.715	278	88.462
168	53.479	205	65.256	242	77.033	279	88.810
169	53.798	206	65.574	243	77.351	280	89.129
170	54.116	207	65.893	244	77.670	281	89.447
171	54.434	208	66.211	245	77.988	282	89.765
172	54.752	209	66.529	246	78.306	283	90.084
173	55.071	210	66.848	247	78.625	284	90.402
174	55.389	211	67.166	248	78.943	285	90.720
175	55.707	212	67.484	249	79.261	286	91.038
176	55.026	213	67.803	250	79.580	287	91.357
177	55.344	214	68.121	251	79.898	288	91.675
178	56.662	215	68.439	252	80.216	289	91.993
179	56.980	216	68.757	253	80.534	290	92.312
180	57.299	217	69.075	254	80.853	291	92.630
181	57.617	218	69.394	255	81.171	292	92.948
182	57.935	219	69.712	256	81.489	293	93.267
183	58.253	220	70.031	257	81.808	194	93.585
184	58.572	221	70.349	258	82.126	295	93.903
185	58.890	222	70.667	259	82.444	296	94.222
186	59.209	223	70.985	260	82.763	297	94.540
187	59.527	224	71.304	261	83.081	298	94.858
188	59.845	225	71.622	262	83.399	299	95.177
189	60.163	226	71.941	263	83.717	300	95.495
190	60.482	227	72.258	264	84.038		

CENTRE OF GRAVITY.

THE centre of Gravity of a body is that point, which, if sustained, the body remains at rest; the particles of which it is composed being equipoised, and having their weights collected, as it were, into that point.

Bodies are reciprocal to each other as their distances from the centre of gravity.—Suppose a rod 11 inches long with a weight of 2 lbs hung at the one end, and a weight of 20 lbs hung at the other end, the centre of gravity, or the point on which this rod so loaded, will balance itself, is just 1 inch from the greater weight, and 10 inches from the less, because, $20 \times 1 = 20$, and $2 \times 10 = 20$; therefore, their weights are inversely as their distances from the centre of gravity.—Hence, the method to find the common centre of gravity of any number of bodies, is, first find the centre between two bodies, then the centre between that centre, and a third body, and so on for a fourth, fifth, &c.; the last centre found being the common centre of all the bodies.

From the foregoing it will easily be conceived, that if a homogeneous beam is balanced upon a

point, that point will be the centre of gravity, and also the centre of the beam; but suppose the beam 10 feet long, each foot weighing 8 lbs, and a weight of 90 lbs suspended from the one end, at what point of the beam will the centre of gravity be?

10 feet, length of beam.—8 lbs each foot in length.
90 lbs weight suspended.

$$\frac{8 \times 10 + 2 \times 90}{8 \times 10 + 90} \times \frac{10}{2} = \frac{260}{170} \times 5 = 7.65$$

+ 2.35 = 10 feet length of beam, that is, the centre of gravity is 2.35 feet from the end at which the weight of 90 lbs is suspended, and will be 7.65 feet from the other end.

Suppose another homogeneous beam, 12 feet long, with a weight of 100 lbs fixed at one end, it is found that the whole is in equilibrio when the beam is suspended 2 feet from the end next the weight; what is the weight of the beam?

100 lbs weight suspended.

2 feet distance from the weight.

10 feet distance from the other end:

$$\frac{2 \times 100 \times 2}{100 - 4} = \frac{400}{96} = 4.166 \text{ lbs the weight of}$$

1 foot of beam, and $4.166 \times 12 = 49.992$ lbs, the weight of the beam.

It is well known to every practical Mechanic, that there are no homogeneous beams or bars:—that it is impossible to find the weight of a foot of length, in a piece of wood, iron, stone, &c. and that the exact

centre of gravity of such materials cannot be found by any known theorem. To obviate this difficulty, and ascertain the true centre of gravity, the beams, bars, &c. are balanced over a prop; but there are many large unwieldy bodies that cannot be thus treated, and for this reason the following data are given, which ascertain nearly the centre required; the data being taken, which are nearest the form and distribution of weight over the body, of which the the centre of gravity is required.

1. The centre of gravity of a triangle is in the straight line, drawn from any angle to the bisection of the opposite side, at the distance of $\frac{2}{3}$ of that line from the angle.

This rule is also true with regard to a pyramid of any number of sides; also to a cone.

2. The centre of gravity of a segment of a circle, is in the radius which bisects it; and its distance from the centre of the circle, is $\frac{1}{2}$ of the cube of its cord divided by the area of the segment.

3. The centre of gravity of a sector of a circle is in the radius which bisects it; and its distance from the centre of the circle, is a fourth proportional to the arc, its chord, and $\frac{2}{3}$ of the radius.

For further information in this article, see *Hutton's Mathematics*, *Banks on Mills*, *Venturoli's Mechanics* by *Cresswell*, &c.

CENTRE OF PERCUSSION.

THE centre of Oscillation or Percussion, is the point in a body revolving round a fixed axis, so taken, that when it is stopped by any force, the whole motion, and tendency to motion, of the revolving body, is stopped at the same time.

It is also that point of a revolving body, which would strike any obstacle with the greatest effect; and from this property, it has received the name of percussion.

The centres of oscillation and percussion are generally treated separately; but the two centres are in the same point, and therefore their properties are the same.

As in bodies at rest, the whole weight may be considered as collected in the centre of gravity; so in bodies in motion, the whole force may be considered as concentrated in the centre of percussion:— therefore, the weight of the rod multiplied by the distance of the centre of gravity from the point of suspension, will be equal to the force of the rod divided by the distance of the centre of percussion from the same point. For example, the length of a rod being 20 feet, and the weight of a foot in length equal to 100 oz.; also a weight or ball fixed

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at under end weighing a 1000 oz.; at what point of the rod from the point of suspension, will be the centre of percussion?*

The weight of the rod is $20 \times 100 = 2000$ oz. which multiplied by half its length $2000 \times 10 = 20000$, gives the momentum of the rod. The weight of the ball = 1000 oz. multiplied by the length of rod = 1000×20 , gives the momentum of the ball. Now the weight of the rod multiplied by the square of the length, and divided by 3 = $\frac{2000 \times 20^2}{3} = 266666$, = the force of the rod,

and the weight of the ball multiplied by the square of the length of the rod, $1000 \times 20^2 = 400000$, is the force of the ball:—therefore the centre of

percussion = $\frac{266666 + 400000}{20000 + 20000} = \frac{666666}{40000} = 16.66$ feet.

For another example; suppose a rod 12 feet long, and 2 lbs. each foot in length, with 2 balls of 3 lbs. each, one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$12 \times 2 \times 6$	$= 144$	momentum of rod.		
3×6	$= 18$	do.	of 1st. ball.	
3×12	$= 36$	do.	of 2d. do.	
	198			

*a = 20 feet long.

b = 100 oz. wt. of a foot in length.

c = 1000 do. fixed at end.

} $\frac{\frac{1}{2}ab \times a^2 + ca^2}{\frac{1}{2}ab \times a + ac} =$ cen-
tre of percussion.

$$\frac{24 \times 144}{3} = 1152, \text{ force of rod.}$$

$$3 \times 36 = 108 \text{ do. of 1st. ball.}$$

$$3 \times 144 = \underline{432} \text{ do. of 2d. ball.}$$

$$1692$$

therefore the centre of percussion = $\frac{1692}{198} = 8.54\ddot{5}$

feet from the point of suspension.

As the centre of percussion is the same with the centre of gravity in the non-application to practical purposes, the following is the easiest and simplest mode of finding it in any beam, bar, &c.

Suspend the body very freely by a fixed point, and make it vibrate in small arcs, counting the number of vibrations it makes in any time, as a minute, and let the number of vibrations made in a minute be called n ; then shall the distance of the centre of oscillation from the point of suspension

be $\frac{140850}{n^2} =$ inches.—For the length of the pendu-

lum vibrating seconds, or 60 times in a minute, being $39\frac{1}{8}$ inches; and the lengths of the pendulums being reciprocally as the square of the number of vibrations made in the same time:—therefore,

$n^2 : 60^2 :: 39\frac{1}{8} : \frac{60^2 \times 39\frac{1}{8}}{n^2} = \frac{140850}{n^2}$ being the

length of the pendulum which vibrates n times in a minute, or the distance of the centre of oscillation below the axis of motion.

There are many situations in which bodies are placed, that prevent the application of the above

rule; and for this reason the following data are given, which will be found useful when the bodies and the forms here given correspond.

1. If the body is a heavy straight line of uniform density, and is suspended by one extremity, the distance of its centre of percussion is $\frac{2}{3}$ ds of its length.

2. In a slender rod of a cylindrical or prismatic shape, whose breadth is very small compared with its length, has the distance of its centre of percussion nearly $\frac{2}{3}$ ds of its length from the axis of suspension.

If these rods were formed so that all the points of their transverse sections were equidistant from the axis of suspension, the distance of the centre of percussion would be exactly $\frac{2}{3}$ ds of their length.

3. In an Isosceles Triangle, suspended by its apex, and vibrating in a plane perpendicular to itself, the distance of the centre of percussion is $\frac{2}{3}$ ths of its altitude. A line or rod, whose density varies as the distance from its extremity, or the point of suspension; also a *fly-wheel*, or *wheels in general*, is in precisely the same predicament as the Isosceles Triangle; *i. e.* the centre of percussion is distant from the centre of suspension $\frac{2}{3}$ ths of its length.

4. In a very slender cone or pyramid, vibrating about its apex, the distance of its centre of percussion is nearly $\frac{2}{3}$ ths of its length.

See *Edinburgh Ency. Article, Centre of Percussion.*

CENTRE OF GYRATION.

THE centre of Oscillation already described, is the point into which all the matter of a body is collected, when the body is put in motion by its own gravity; and the centre of Gyration is the point into which all the matter of a body is collected, when it is put in motion by any extraneous force.

If a straight bar, equally thick, was struck at the centre of gyration, the stroke would communicate the same angular velocity to the bar, as if the whole bar was collected in that point.

The force of any particle revolving round a centre, is, as that particle multiplied by the square of its velocity, or of its distance from the centre of motion; consequently, the force required to destroy that motion must be equal to it.

For example; suppose a bar of an uniform density 12 feet long, and each foot weighing 7 lbs, and revolving upon a centre 3 feet from the one end; at what distance will the centre of gyration be from the centre of motion?

$a = 9$ feet long end.

$b = 3$ do. short end.

$c = 7$ lbs. each foot.

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$$\begin{aligned}
 9 \times 7 &= 63, \text{ weight of long end.} \\
 3 \times 7 &= 21, \text{ weight of short end.} \\
 63 \times 9^2 &= 5103, \text{ force of long end.} \\
 21 \times 3^2 &= 189, \text{ do. of short end.}
 \end{aligned}$$

Centre of gyration $\sqrt{\frac{5103+189}{3 \times 63+21}} = 4.5$ feet from
centre of motion.*

For another example—Let the same bar have a weight of 50 lbs at each end, then at what distance will the centre of gyration be from the centre of motion?

$$\begin{aligned}
 a &= 9 \text{ feet, long end.} \\
 b &= 3 \text{ do. short end.} \\
 c &= 7 \text{ lbs each foot.} \\
 d &= 50 \text{ lbs weight at long end.} \\
 e &= 50 \text{ lbs weight at short end.} \\
 z &= \text{distance of centre of gyration from centre of} \\
 &\quad \text{motion.}
 \end{aligned}$$

$$\begin{aligned}
 63 \times 9^2 &= 5103, \text{ force of long end.} \\
 21 \times 3^2 &= 189, \text{ do. of short end.} \\
 50 \times 9^2 &= 4050, \text{ do. of weight on long end.} \\
 50 \times 3^2 &= 450, \text{ do. of weight on short end.}
 \end{aligned}$$

$$\frac{*ac.a^2}{3} + \frac{bc.b^2}{3} = \frac{ac.a^2 + bc.b^2}{3ac + bc} \text{ or } z = \sqrt{\frac{ac.a^2 + bc.b^2}{3ac + bc}} \text{ } z \text{ being the centre of gyration.}$$

$$\sqrt{\frac{3 \times 4050 + 450 + 5103 + 189}{3 \times 50 + 50 + 63 + 21}} = \sqrt{\frac{18792}{552}} = 5.84$$

distance between the centres.*

The centre of gyration, with respect to practical utility, is the same as the two foregoing centres. The following Rule is the easiest mode of ascertaining the centre of gyration.

“ If the distance of the centre of oscillation from the centre of the system, or point of suspension, be multiplied by the distances of the centre of gravity from the same point, the square root of the product will be the distance of the centre of gyration; *i. e.* let the centre of gravity be 4, and the centre of oscillation 9, then $4 \times 9 = 36$, and the square root of that is 6; therefore 6 is the distance that the centre of gyration is from the point of suspension.

* $da^2 + \frac{ca^3}{3} + eb^2 + \frac{cb^3}{3} =$ the force of the whole revolving round the centre of motion, and which must be equal to $\frac{ac + d + bc + e}{3} \times Z^2$, therefore

$$Z^2 \times \frac{ac + d + bc + e}{3} = da^2 + eb^2 + \frac{ca^3 + cb^3}{3}, \text{ or}$$

$$Z^2 = \frac{3da^2 + eb^2 + ca^3 + cb^3}{3ac + d + bc + e}, \text{ and}$$

$$Z = \sqrt{\frac{3da^2 + eb^2 + ca^3 + cb^3}{3ac + d + bc + e}}$$

The following note is given by Dr. Hutton, in the 3d vol. of his *Math. art. max. of Machines*, p. 244.

The distance of *r*, the centre of gyration, from *c* the centre or axis of motion, in some of the most useful cases, is as below.

In a circular wheel of uniform thickness	}	$CR = \text{rad.} \sqrt{\frac{1}{2}}$.
In the periphery of a circle revolving about the diameter	}	$CR = \text{rad.} \sqrt{\frac{1}{4}}$.
In the plane of a circle ditto		$CR = \frac{1}{2} \text{rad.}$
In the surface of a sphere ditto		$CR = \text{rad.} \sqrt{\frac{8}{3}}$.
In a solid sphere . . . ditto		$CR = \text{rad.} \sqrt{\frac{5}{3}}$.
In a plane ring formed of circles whose radii are <i>R, r</i> , revolving about centre	}	$CR = \sqrt{\frac{R^4}{2R^2 - 2r^2}}$.
In a cone revolving about its vertex		$CR = \frac{1}{2} \sqrt{\frac{1}{3} a^2 + \frac{1}{3} r^2}$.
In a cone its axis		$CR = r \sqrt{\frac{5}{15}}$.
In a straight lever whose arms are <i>R</i> and <i>r</i>	}	$CR = \sqrt{\frac{R^5 + r^5}{3(R+r)}}$.

SUMMARY.

If *P* be any particle of a body *B*, and *d* its distance from the axis of motion *s*, also *G O R* the centres of Gravity, Oscillation, and Gyration. Then the centres of

Gravity will be = $\frac{Pd}{B} = G.$

Percussion do. = $\frac{Pd^2}{SGB} = O.$

Gyration do. = $\sqrt{\frac{Pd^2}{B}} = R.$

For ample Explanations and Examples of the foregoing Centres, See *Hutton's Mathematics, Banks on Mills, &c.*

CENTRAL FORCES.

1. The quantity of matter in a body is as its magnitude and density; that is, if a body measures 7 cubic feet, and a cubic foot weighs 10 lbs, the quantity of matter in that body will be $= 7 \times 10 = 70$ lbs.

2. All bodies naturally endeavour to continue in their present state, whether of rest or motion.

3. When a body at rest is struck by a force so as to produce motion, that motion is in proportion to the force, and in the direction of the right line in which it acts.

4. Action and Reaction between any two bodies, are equal and contrary; that is, by Action and Reaction equal changes of motion are produced in bodies acting on each other; and these changes are directed towards opposite or contrary parts.

5. The Momenta, or quantities of motion in moving bodies, are as their masses and velocities; for a body of 20 lbs, moving at a velocity of 10 feet, will have a momentum of 200; but a body of 6 lbs, moving at the same velocity, will have only 60 for its momentum.

6. A body moving round a central point inclines to fly off in a straight line, from the first impulse of

motion; the force which causes it to leave that line, or move in a circle round the point, is called the Centripetal; and the resistance which it affords, the Centrifugal force; or, in other words, when a body revolves round its centre of motion, the centrifugal force is that power or tendency which the body has to burst or fly asunder; and the centripetal force, is that power which keeps the body from bursting or flying asunder.

It is evident from the last remark, that the greater the velocity, the greater will be the centrifugal force; and from the 5th remark, the greater the mass the greater the momentum; therefore, as is the weight and velocity of the revolving body, so is the centrifugal force.

Suppose two fly-wheels of the same weight, one of them 12 feet diameter, and revolving in 8 seconds; what must be the diameter of the other, when it revolves in 3 seconds?

The diameter and velocity of the first, must be equal to the diameter and velocity of the second; therefore $8^2 : 12 :: 3^2$ is to the diameter $= \frac{12 \times 3^2}{8^2}$
 $= \frac{108}{64} = 1.6875$ foot, the diameter of the second fly-wheel at the circle of percussion.

Again, suppose two fly-wheels of the same diameter, the one revolving in 3 seconds, and the other in 8 seconds; what will be the difference of their weights?

As $3^2 : 8^2$ so is the weight of the one, to the weight of the other.

$\frac{8^2}{3^2} = \frac{64}{9} = 7\frac{1}{9}$, their weights will be to each other as $7\frac{1}{9}$ is to 1; and by knowing the weight of the second, and dividing it by $7\frac{1}{9}$, will give the weight of the first.

In the two preceding Examples, weight and velocity are taken separately.—The following Examples give the centrifugal force, when both weight and velocity are used.

Required the centrifugal force of a fly-wheel, diameter 16 feet, velocity 50 revolutions per minute, and weight $3\frac{1}{2}$ tons?

$3.1416 =$ circumference of a circle the diameter 1.
16 feet = space a body falls through in 1 second of time.

$.833 =$ time of one revolution.

$$\frac{16 \times 3.1416^2}{16 \times .833^2} = \frac{157.9136}{11.1122} = 14.21 \text{ times the weight}$$

in tons, the wt. being $3\frac{1}{2}$ tons; therefore $3.5 \times 14.21 = 49.73$ tons, the centrifugal force.

The stones on which they grind table-knives at Sheffield, are about 44 inches diameter, and weigh about half a ton; the velocity of the circumference is at the rate of 1250 yards in a minute, equal to 326 revolutions; required the centrifugal force?

$22 \times 2 = 968$, the square root of which is 31.1 inches, or 2.59 feet, the diameter of the circle of gyration.

As $326 : 60 :: 1 : .184$ seconds, the time of one revolution.

$$\frac{2.59 \times 3.1416^2}{16 \times 184^2} = \frac{25.5622}{.54169} = 47.18 \text{ times the}$$

weight of the stone: the stone is .5 ton, therefore $47.18 \times .5 = 23\frac{1}{2}$ tons centrifugal force.

BANKS.

MOTION, RESISTANCE, AND EFFECT OF MACHINES.

VARIOUS as the modifications of Machines are, and innumerable their different applications; still there are only three distinct objects to which their utility tends.

The first is, in furnishing the means of giving to the moving force the most commodious direction; and when it can be done, of causing its action to be applied immediately to the body to be moved. These can rarely be united; but the former can be accomplished in most cases.

The second, in accommodating the velocity of the work to be performed, to the velocity with which alone a natural power can act.

The third, and most essential advantage of machines, is in augmenting, or rather in modifying the energy of the moving power, in such a manner,

that it may produce effects of which it would have been otherwise incapable. For instance, a man might with exertion lift 4 cwt.; but let him apply a lever, and he will lift many times that weight.

The motions produced by machines are of three kinds, viz. Accelerated, Uniform, and Alternate, *i. e.* accelerated and retarded. The first of these always takes place when the moving power is immediately applied; the second, after the machine has been in motion for a short time; the third, in intermitting machines, such as pendulum clocks, &c.; but though a second's pendulum is accelerated the first half second, and retarded the next; still it produces a constant number of vibrations in a given time, and therefore may be considered as a machine of uniform motion.

The grand object in all practical cases, is, to procure a uniform motion, because it produces the greatest effect. All irregularities of motion indicate that there is some point resisting the motion, and to overcome which a part of the propelling power is wasted, and the greatest varying velocity is only equal to that velocity by which the machine would move when its motion is uniform. If the machine moves with an accelerating velocity, it is certain that the power is greater than what balances the opposing resistance, and therefore cannot produce the greatest effect; because the whole resistance is not applied. In both these cases the machine has neither the power nor the effect which it would have if moving uniformly.

When irregularity of motion takes place, particularly in a large heavy machine, it suffers a continual straining and jolting, which must very soon destroy it. It is therefore of the greatest consequence, that, from all machines, every cause tending to produce irregularity of motion should be taken away.

Many fundamental Rules having already been given; and from what has just now been stated respecting the motion of machines, it is thought unnecessary to give any elaborate calculation on the maximum and minimum effects of motion; therefore the following statements of the effects of one horse power, in the different applications enumerated, will conclude this article.

One horse's power, at a maximum, is calculated to lift, by means of a pump, 250 hhds. of water, 10 feet high in one hour.

One horse's power, at a medium, is calculated to drive 100 spindles, with preparation, of cotton yarn twist.

Ditto ditto, 500 spindles, with preparation, of Mule yarn, No. 48.

Ditto ditto, 1000 spindles, with preparation, of Mule yarn, No. 110.—The intermediate numbers in proportion.

Ditto ditto, 12 power looms, with preparation.

STEAM ENGINE.

THE Rules of Practical Mechanics, with various Examples, have been stated, embracing the Mechanical Powers, which exhibit the Analysis of Machinery—The Rules for finding the weight and strength of the different parts—The centres of Gravity, Percussion, and Gyration, which determine, according to the nature of the Machine, that point in which all its force is collected—And an explanation of the nature of Motions, and the effect of Machines; showing, that when a Machine is producing the greatest effect, or working at a maximum, that the motion is uniform; the power and resistance being in a proper proportion to each other.

What remains now to be explained, are, the Rules connected with the Steam Engine, Water Wheel, Common and Force Pumps, which are, *viz.*

STEAM ENGINE.

FUEL.—To produce equal heats, $\frac{3}{4}$ cwt. of Newcastle Coal is equal to 1 cwt. of Glasgow Coal, and to $2\frac{1}{4}$ cwt. of Wood, or three times the weight. Also, it takes double the weight of Culm to that of Coal.

Upon the premises of Messrs. Claud Girdwood & Co. of Glasgow, there are two Steam Engines, one of 32 horse power, and one of 12 horse power. The daily consumpt of Culm for the first, upon an average, is 5 waggons, 24 cwt. each; for the second, $1\frac{1}{2}$ waggon. Taking an average from these two, gives 34 lbs of Culm per hour for each horse power.

The consumpt of Coal per hour for one horse power, is from 13 to 20 lbs, according to the strength of the fuel, and the manner by which the furnace is fed.—Consumpt of Wood per hour for one horse power, is 50 lbs.—Consumpt of Culm per hour for one horse power, is 34 lbs.

BOILERS—are of various forms, but the most general is proportioned as follows, viz. width 1, depth 1.1, length 2.5; their capacity being, for the most part, two horse more than the power of the Engine for which they are intended.

Boulton and Watt allow 25 cubic feet of space for each horse power, some of the other Engineers allow 5 feet of surface of water.

STEAM—arising from water at the boiling point, is equal to the pressure of the atmosphere, which is in round numbers, 15 lbs on the square inch; but to allow for condensation, and escapes, the safety valve of a boiler is loaded generally with 3 lbs upon the square inch, which makes the pressure in the boiler 18 lbs on the square inch of surface.

The following Table shows the pressure of Steam, according to its heat and expansion.

Steam predominating over the pressure of the atmosphere, upon a safety valve, if its elastic force be equal to	Libs.	Degrees.	degrees of heat by Fahrenheit, and at these respective degrees of heat, steam can expand to about	Times its volume
	5	227		5
	6	230		6
	7	232		7
	8	235		8
	9	237		9
10	15	239		10
15	20	250		15
20	25	259		20
25	30	267		25
30	35	273		30
35	40	278		35
40		282		40

By small additions to the temperature, an expansive force may be given to steam so as to be equal to 400 times its natural bulk, or in any other proportion, provided the vessels, &c. that contain it be strong in proportion.

The following is a simple Rule for finding the quantity of steam required to raise a given quantity of water to any given temperature.

RULE. Multiply the water to be warmed by the difference of temperature between the cold water and that to which it is to be raised, for a dividend; then to the temperature of the steam add 900 degrees, and from that sum take the required temperature of the water: this last remainder being made a divisor to the above dividend, the quotient will be the quantity of steam in the same terms as the water.

EXAMPLE.

What quantity of steam at 212° will raise 100 gallons of water at 60° up to 212° ?

$$\frac{212^{\circ} - 60^{\circ} \times 100}{212^{\circ} + 900^{\circ} - 212^{\circ}} = 17 \text{ gallons of water formed into steam.}$$

Now steam at the temperature of 212° is 1800 times its bulk in water; or 1 cubic foot of steam, when its elasticity is equal to 30 inches of mercury, contains 1 cubic inch of water.—Therefore 17 gallons of water converted into steam, occupies a space of $4090\frac{1}{2}$ cubic feet, having a pressure of 15 lbs on the square inch.

HORSE POWER.—Boulton and Watt suppose a horse able to raise 32,000 lbs avoirdupois 1 foot high in a minute.

Desaguliers makes it 27,500 lbs.

Smeaton do. 22,916 do.

Boulton and Watt, however, in calculating the power of their Engines, suppose a horse to draw 200 lbs at the rate of $2\frac{1}{2}$ miles an hour, or 220 feet per minute, with a continuance, drawing the weight over a pulley—now, $200 \times 220 = 44000$, *i. e.* 44000 lbs at 1 foot per minute, or 1 lib at 44000 feet per minute.

LENGTH OF STROKE.—The stroke of an Engine is equal to one revolution of the crank shaft, therefore double the length of the cylinder. When stating the length of stroke, the length of cylinder is only

given, that is, an Engine with a 3 feet stroke, has its cylinder 3 feet long, besides an allowance for the piston.

The following Table shews the length of stroke, (or length of cylinder), and the number of feet the piston travels in a minute, according to the number of strokes the Engine makes when working at maximum.

Length of Stroke.	Number of Strokes.	Feet $\frac{P}{M}$ Minute.
Feet 2	43	172
" 3	32	192
" 4	25	200
" 5	21	210
" 6	19	228
" 7	17	238
" 8	15	240
" 9	14	250

CYLINDER.—The pressure of the steam is 15 lbs on every square inch of the area of cylinder; but the piston falls and rises with only a force of 10 lbs on the square inch, thus $\frac{1}{3}$ d of the power is lost by being required to overcome the friction of the Engine.

To calculate the power of the Engine.

RULE. Multiply the area of cylinder by the effective pressure = 10 lbs, the product is the weight the Engine can raise.—Multiply this weight by the number of feet the piston travels in one minute, which will give the momentum, or weight,

the Engine can lift 1 foot high per minute; divide this momentum by a horse power, as previously stated, and the quotient will be the number of horse power the Engine is equal to.

EXAMPLE I.

What is the power of an Engine, the cylinder being 42 inches diameter, and stroke 5 feet?

$$\frac{42^2 \times .7854 \times 10 \times 210}{44000} = 66.12 \text{ horse power.}$$

EXAMPLE II.

What size of cylinder will a 60 horse power Engine require, when the stroke is 6 feet?

$$\frac{44000 \times 60}{228 \times 10} = 1158 \text{ inch. area of cylinder.}$$

Note. To find the power to lift a weight at any velocity, multiply the weight in lbs by the velocity in feet, and divide by the horse power; the quotient will be the number of horse power required.

NOZLES.—The diameter of the valves of Nozles ought to be fully one-fifth of the diameter of cylinder.

AIR-PUMP.—The solid contents of the Air-Pump is equal to the fourth of the solid contents of cylinder, or when the Air-pump is half the length of the stroke of the Engine, its area is equal to half the area of cylinder.

CONDENSER—is generally equal in capacity to the Air-pump, but when convenient, it ought to be more; for when large there is a greater space of vacuum, and the steam is sooner condensed.

COLD WATER PUMP.—The capacity of the Cold Water Pump depends on the temperature of the water. Many Engines return their water, which cannot be so cold as water newly drawn from a river, well, &c.; but when water is at the common temperature, each horse power requires nearly $7\frac{1}{2}$ gallons per minute:—taking this quantity as a standard, the size of the pump is easily found by the following Rule, viz.—Multiply the number of horse power by $7\frac{1}{2}$ gallons, and divide by the number of strokes per minute; this will give the quantity of water to be raised each stroke of pump.—Multiply this quantity by 231, (the number of cubic inches in a gallon), and divide by the length of effective stroke of pump, the quotient will be the area.

EXAMPLE.

What diameter of pump is requisite for a 20 horse power Steam Engine, having a 3 feet stroke, the effective stroke of pump to be 15 inches?

$20 \times 7\frac{1}{2} = \frac{150}{32} = 4.6875$ gallons the pump lifts each stroke.

$\frac{4.6875 \times 231}{15} = 72.1875$ inches area of pump.

HOT WATER PUMP.—The quantity of water raised at each stroke ought to be equal in bulk to the 900th part of the capacity of the cylinder.

PROPORTIONS.—The length of stroke being 1, the length of beam to centre will be 2, the length of crank .5, and the length of connecting rod 3.

The following is a Table showing the force which the connecting rod has to turn round the crank at different parts of the motion.

	A	B	C	D
<i>Col. A.</i> Decimal proportions of descent of the Piston, the whole descent being 1.	.0	180°	.0	.0
	.05	151½	.46	.128
	.10	141	.62	.158
	.15	131½	.74	.228
<i>Col. B.</i> Angle between the connecting Rod and Crank.	.2	123	.830	.271
	.25	117	.892	.308
	.3	110½	.94	.342
	.35	104	.976	.377
<i>Col. C.</i> Effective length of the Lever upon which the connecting Rod acts, the whole Crank being 1.	.4	97½	.986	.41
	.45	91½	1.	.441
	.5	85½	1.	.473
	.55	80	.986	.507
<i>Col. D.</i> Decimal proportions of half a revolution of the Fly-Wheel.	.6	75	.956	.538
	.65	69	.92	.572
	.7	62½	.88	.607
	.75	57½	.824	.642
<i>Col. C.</i> also shews the force which is communicated to the Fly Wheel, expressed in decimals, the force of the Piston being 1.	.8	49	.746	.68
	.85	42	.66	.723
	.9	34	.546	.776
	.95	23½	.390	.84
	1.0	0	.000	1.0

FLY-WHEEL—Is used to regulate the motion of the engine, and to bring the crank past its centres. The Rule for finding its weight, is,—Multiply the

number of horses' power of the Engine by 2000, and divide by the square of the velocity of the circumference of the wheel per second, the quotient will be the weight in cwts.

EXAMPLE.

Required the weight of a fly-wheel proper for an Engine of 20 horse-power, 18 feet diameter, and making 22 revolutions per minute?

18 feet diameter = 56 feet circumference, \times 22 revolutions per minute = 1232 feet, motion per minute \div 60 = $20\frac{1}{2}$ feet motion per second; then $20\frac{1}{2}^2 = 420\frac{1}{4}$ the divisor.

20 horse power \times 2000 = 40000 dividend.

$$\frac{40000}{420\frac{1}{4}} = 90.4 \text{ cwt. weight of wheel.}$$

PARALLEL MOTION.—The radius and parallel bars are of the same dimensions; their length being generally 1-4th of the length of the beam between the two glands, or one-half of the distance between the fulcrum and gland. Both pairs of straps are the same length between the centres, and which is generally three inches less than the half of the length of stroke.

GOVERNOR, OR DOUBLE PENDULUM.—If the revolutions be the same, whatever be the length of the arms, the balls will revolve in the same plane, and the distance of that plane from the point of suspension, is equal to the length of a pendulum, the vibrations of

which will be double the revolutions of the balls. For example; suppose the distance between the point of suspension and plane of revolution be 36 inches, the vibrations that a pendulum of 36 inches will make per minute, is $= \frac{375}{\sqrt{36}} = 62$ vibrations, and $\frac{62}{2} = 31$ revolutions per minute the balls ought to make.

WATER WHEEL.

THIS subject belongs to Hydrodynamics, also the common and force Pumps; and since they are the last of this Treatise, they may be classed under that name, to distinguish them from the preceding subjects in Statics and Dynamics.

WATER. (*Hydrostatics.*)

Hydrostatics is the science which treats of the pressure, or weight, and equilibrium of water, and other fluids, especially those that are non-elastic.

Note 1. The pressure of water at any depth, is as its depth; for the pressure is as the weight, and the weight is as the height.

Note 2. The pressure of water on a surface any how immersed in it, either perpendicular, horizontal, or oblique, is equal to the weight of a column of water, the base being equal to the surface pressed, and the altitude equal to the depth of the centre of gravity, of the *surface pressed*, below the top or surface of the fluid.

PROBLEM I.

In a vessel filled with water, the sides of which are upright and parallel to each other, having the top of the same dimensions as the bottom, the pressure exerted against the bottom, will be equal to the area of the bottom multiplied by the depth of water.

EXAMPLE.

A vessel, 3 feet square and 7 feet deep, is filled with water; what pressure does the bottom support?

$$\frac{3^2 \times 7 \times 1000}{16} = 3937\frac{1}{2} \text{ lbs Avoirdupois.}$$

PROBLEM II.

A side of any vessel sustains a pressure equal to the area of the side multiplied by half the depth, therefore the sides and bottom of a cubical vessel sustain a pressure equal to three times the weight of water in the vessel.

EXAMPLE I.

The gate of a sluice is 12 feet deep and 20 feet broad; what is the pressure of water against it?

$$\frac{20 \times 12 \times 6 \times 1000}{16} = 90000 = 40\frac{1}{2} \text{ tons nearly.}$$

K

From Note 2d.—The pressure exerted upon the side of a vessel, of whatever shape it may be, is as the area of the side and centre of gravity below the surface of water.

EXAMPLE II.

What pressure will a board sustain, placed diagonally through a vessel, the side of which is 9 feet deep, and bottom 12 feet by 9 feet?

$\sqrt{12^2 + 9^2} = 15$ feet, the length of diagonal board.

$$\frac{15 \times 9 \times 4\frac{1}{2} \times 1000}{16} = 37969 \text{ lbs nearly.}$$

Though the diagonal board bisects the vessel, yet it sustains more than the half of the pressure in the bottom, for the area of bottom is 12×9 , and the half of the pressure is $\frac{1}{2}60750 = 30375$.

The bottom of a conical or pyramidal vessel sustains a pressure equal to the area of the bottom and depth of water, consequently, the excess of pressure is three times the weight of water in the vessel.

WATER. (*Hydraulics.*)

Hydraulics is that science which treats of fluids considered as in motion, it therefore embraces the phenomena exhibited by water issuing from orifices in reservoirs, projected obliquely, or perpendicularly, in *Jet-d'eau*, moving in pipes, canals, and rivers, oscillating in waves, or opposing a resistance to the progress of solid bodies.

It would be needless here to go into the minutiae of hydraulics, particularly when the theory and practice do not agree. It is only the general laws, deduced from experiment, that can be safely employed in the various operations of hydraulic architecture.

Mr. Banks, in his Treatise on Mills, after enumerating a number of experiments on the velocity of flowing water, by several philosophers, as well as his own, takes from thence the following simple rule, which is as near the truth as any that have been stated by other experimentalists.

RULE. Measure the depth (of the vessel, &c.) in feet, extract the square root of that depth, and multiply it by 5.4, which gives the velocity in feet per second; this multiplied by the area of the orifice in feet, gives the number of cubic feet which flows out in one second.

EXAMPLE.

Let a sluice be 10 feet below the surface of the water, its length 4 feet, and open 7 inches; required the quantity of water expended in one second?

$$\sqrt{10} = 3.162 \times 5.4 = 17.0748 \text{ feet velocity.}$$

$$\frac{4 \times 7}{12} = 2\frac{1}{3} \text{ feet} \times 17.0748 = 39.84 \text{ cubic feet of water per second.}$$

If the area of the orifice is great compared with the head, take the medium depth, and two-thirds of the velocity from that depth, for the velocity

EXAMPLE.

Given the perpendicular depth of the orifice 2 feet, its horizontal length 4 feet, and its top 1 foot below the surface of water. To find the quantity discharged in one second:

The medium depth is $= 1.5 \times 5.4 = 8.10 - 3 = 5.40 \times 8 = 43.20$ cubic feet.*

The quantity of water discharged through slits, or notches, cut in the side of a vessel or dam, and open at the top, will be found by multiplying the velocity at the bottom by the depth, and taking $\frac{2}{3}$ of the product for the area; which again multiplied by the breadth of the slit, or notch, gives the quantity of cubic feet discharged in a given time.

EXAMPLE.

Let the depth be 5 inches, and the breadth 6 inches; required the quantity run out in 46 seconds?

The depth is .4166 of a foot.

The breadth is .5 of a foot.

$\sqrt{.4166} = .6455 \times 5.4 \times \frac{2}{3} = 2.3238 \times .4166 = 96825 \times 5 = .48412$ feet per second.

Then $.48412 \times 46. = 22.269$ cubic feet in 46 seconds.

There are two kinds of water wheels, Undershot and Overshot. Undershot, when the water strikes the wheel at, or below the centre. Overshot, when the water falls upon the wheel above the centre.

* The square root of the depth is not taken in this example, but when the depth is considerable, it ought to be taken.

The effect produced by an *undershot* wheel, is from the impetus of the water. The effect produced by an *overshot* wheel, is from the gravity or weight of the water.

Of an undershot wheel, the power is to the effect as 3 : 1.—Of an overshot wheel, the power is to the effect as 3 : 2—which is double the effect of an undershot wheel.

The following is an *Abridgement of SMEATON on WATER WHEELS.*

UNDERSHOT.

Velocity of water in l''	= V	V.A = Q in one second. $\frac{QW.V}{3} = P$; Power to produce mechanical effect.
Weight of 1 cub. in. of water	= W	
Area of sluice	= A	
Quantity of water	= Q	
Power of the water to produce mechanical effect	= P	

POWER AND EFFECT AT MAXIMUM.

Velocity of wheel in l''	= v	$V - v = E$ $wv = e$ P : e :: 10 : 3.62, or 3 : 1 V : v :: 10 : 3.5, or 5 : 2
Effective velocity of water	= E	
Effect produced by the wheel	= e	
Weight raised	= w	
Velocity of weight raised	= v	

OVERSHOT.

Descent of water including head	} = D	D.W = P.
and diameter of wheel*		
The weight of water expended	} = W	
in one second		

POWER AND EFFECT AT MAXIMUM.

Power of the water is = D.W = P	P : e :: 10 : 6.6, or 3 : 2 nearly. Double that of an Undershot.
Effect of the wheel is = wv = e	

* By Head is understood the distance between the orifice and the part of the wheel on which the water falls. The fall is the perpendicular height from the bottom of wheel to the orifice.

The velocity at a maximum is = 8 feet in one second.

Since the effect of the overshot is double that of the undershot, it follows that the higher the wheel is in proportion to the whole descent, the greater will be the effect.

The maximum load for an overshot wheel, is that which reduces the circumference of the wheel to its proper velocity, = 8 feet in 1 second; and this will be known, by dividing the effect it ought to produce in a given time, by the space intended to be described by the circumference of the wheel in the same time; the quotient will be the resistance overcome at the circumference of the wheel, and is equal to the load required, the friction and resistance of the machinery included.

The following is an Extract from Banks on Mills, page 152.

“ The effect produced by a given stream in falling through a given space, if compared with a weight, will be directly as that space; but if we measure it by the velocity communicated to the wheel, it will be as the square root of the space descended through, agreeably to the laws of falling bodies.

“ *Experiment 1.* A given stream is applied to a wheel at the centre; the revolutions per minute are 38.5.

“ *Ex. 2.* The same stream applied at the top, turns the same wheel 57 times in a minute.

“ If in the first experiment the fall is called 1, in the second it will be 2: then $\sqrt{1} : \sqrt{2} :: 38.5 : 54.4$, which are in the same ratio as the square roots of the spaces fallen through, and near the observed velocity.

“ In the following experiments a fly is connected with the water wheel.

“ *Ex. 3.* The water is applied at the centre, the wheel revolves 13.03 times in one minute.

“ *Ex. 4.* The water is applied at the vertex of the wheel, and it revolves 18.2 times per minute.

“ As 13.03 : 18.2 :: $\sqrt{1} : \sqrt{2}$ nearly.

“ From the above we infer, that the circumferences of wheels of different sizes may move with velocities which are as the square roots of their diameters without disadvantage, compared one with another, the water in all being applied at the top of the wheel: for the velocity of falling water at the bottom or end of the fall is as the time, or as the square root of the space fallen through: for example, let the fall be 4 feet, then, As $\sqrt{16} : 1'' :: \sqrt{4} : \frac{1}{2}''$, the time of falling through 4 feet:—Again, let the fall be 9 feet, then, $\sqrt{16} : 1'' :: \sqrt{9} : \frac{3}{4}''$, and so for any other space, as in the following Table, where it appears that water will fall through one foot in a quarter of a second, through 4 feet in half a second, through 9 feet in 3 quarters of a second, and through 16 feet in one second. And if a wheel 4 feet in diameter moved as fast as the water, it could not revolve in less than 1.5 second, neither could a wheel of 16 feet diameter revolve in less

than three seconds; but though it is impossible for a wheel to move as fast as the stream which turns it, yet, if their velocities bear the same ratio to the time of the fall through their diameters, the wheel 16 feet in diameter may move twice as fast as the wheel 4 feet diameter."

TABLE.

Height of the fall in Feet.	Time of falling in Seconds.	Height of the fall in Feet.	Time of falling in Seconds.
1	.25	14	.935
2	.352	16	1.
3	.432	20	1.117
4	.5	24	1.22
5	.557	25	1.25
6	.612	30	1.37
7	.666	36	1.5
8	.706	40	1.58
9	.75	45	1.67
10	.79	50	1.76
12	.864		

POWER AND EFFECT.—The power water has to produce mechanical effect, is as the quantity and fall of perpendicular height.—The mechanical effect of a wheel is as the quantity of water in the buckets and the velocity.

The power is to the effect as 3 : 2, that is, suppose the power to be 9000, the effect will be

$$= \frac{9000 \times 2}{3} = \frac{18000}{3} = 6000.$$

HEIGHT OF THE WHEEL.—The higher the wheel is in proportion to the fall, the greater will be the effect, because it depends less upon the impulse,

and more upon the gravity of the water; however, the head should be such, that the water will have a greater velocity than the circumference of the wheel; and the velocity that the circumference of the wheel ought to have, being known, the head required to give the water its proper velocity, can easily be known from the rules of Hydrostatics.

VELOCITY OF THE WHEEL.—Banks, in the foregoing quotation, says, “That the circumferences of overshot wheels of different sizes may move with velocities as the square roots of their diameters, without disadvantage.”—Smeaton says, “Experience confirms that the velocity of 3 feet per second is applicable to the highest overshot wheels, as well as the lowest; though high wheels may deviate further from this rule, before they will lose their power, by a given aliquot part of the whole, than low ones can be admitted to do; for a 24 feet wheel may move at the rate of 6 feet per second, without losing any considerable part of its power.”

It is evident that the velocities of wheels, will be in proportion to the quantity of water and the resistance to be overcome:—if the water flows slowly upon the wheel, more time is required to fill the buckets than if the water flowed rapidly; and whether Smeaton or Banks is taken as a data, the mill-wright can easily calculate the size of his wheel, when the velocity and quantity of water in a given time is known.

EXAMPLE I.

What power is a stream of water equal to, of the following dimensions, viz. 12 inches deep, 22 inches broad; velocity, 70 feet in $11\frac{1}{4}$ seconds, and fall, 60 feet.—Also, what size of a wheel could be applied to this fall?

$$\frac{12 \times 22}{144} = 1.83 \text{ square feet:—area of stream.}$$

$11\frac{1}{4}'' : 70 :: 60'' : 357.5$ lineal feet per min.—velocity.

$357.5 \times 1.83 = 654.225$ cubic feet per minute.

$654.225 \times 62.5 = 40889.0625$ avoird. lbs per minute.

$40889.0625 \times 60 = 2453343.7500$ momentum at a fall of 60 feet.

$$\frac{2453343.7500}{44000} = 55.7 \text{ horse power.}$$

$3 : 2 :: 55.7 : 37.13$ effective power.

The diameter of a wheel applicable to this fall, will be 58 feet, allowing one foot below for the water to escape, and one foot above for its free admission.

$58 \times 3.1416 = 182.2128$ circumference of wheel.

$60 \times 6 = 360$ feet per minute, = velocity of wheel.

$$\frac{654.225}{360} = 1.8 \text{ sectional area of buckets.}$$

The buckets must only be half full, therefore $1.8 \times 2 = 3.6$ will be the area.

To give sufficient room for the water to fill the buckets, the wheel requires to be 4 feet broad,

now, $\frac{3.6}{4} = .9$, say 1 foot depth of shrouding.

$\frac{360}{182.2128} = 1.9$ revolutions per minute the wheel will make.

Power of water . . .	= 55.7	H. P.	}	<i>Ans.</i>	
Effective power of do. =	37.13	H. P.			
Dimensions of Wheel.	{	Diameter . . .			= 58 Feet.
		Breadth . . .			= 4 Feet.
		Depth of shrouding =	1 Foot.		

EXAMPLE II.

What is the power of a water wheel, 16 feet diameter, 12 feet wide, and shrouding 15 inches deep.

$16 \times 3.1416 = 50.2656$ circumference of wheel.
 $12 \times 1\frac{1}{4} = 15$ square feet, sectional area of buckets.
 $60 \times 4 = 240$ lineal feet per minute, = velocity.
 $240 \times 15 = 3600$ cubic feet water, when buckets are full; when half full, 1800 cubic feet.
 $1800 \times 62.5 = 112500$ avoird. lbs of water per minute.
 $112500 \times 16 = 1800000$ momentum, falling 16 feet.
 $3 : 2 :: 1800000 : \frac{1200000}{44000} = 27$ horse power.

BUCKETS.—The number of buckets to a wheel should be as few as possible, to retain the greatest quantity of water; and their mouths only such a width as to admit the requisite quantity of water, and at the same time sufficient room to allow the air to escape.

THE COMMUNICATION OF POWER.—There are no prime movers of machinery from which power is

taken in a greater variety of forms than the water wheel, and among such a number there cannot fail to be many bad applications.

Suffice it here to mention one of the worst, and most generally adopted. For driving a cotton mill in this neighbourhood, there is a water wheel about 12 feet broad, and 20 feet diameter; there is a division in the middle of the buckets upon which the segments are bolted round the wheel, and the power is taken from the vertex: from this erroneous application, a great part of the power is lost; for the weight of water upon the wheel presses against the axle in proportion to the resistance it has to overcome, and if the axle was not a large mass of wood, with very strong iron journals, it could not *stand* the great strain which is upon it.

The most advantageous part of the wheel, from which the power can be taken, is that point in the circle of gyration horizontal to the centre of the axle; because, taking the power from this part, the whole weight of water in the buckets acts upon the teeth of the wheels; and the axle of the water wheel suffers no strain.

The proper connection of machinery to water wheels is of the first importance, and mismanagement in this particular point is often the cause of the journals and axles giving way, besides a considerable loss of power.

To find the radius of the circle of gyration in a water wheel is therefore of advantage to the saving of power, and the following Example will show the rule by which it is found. *See Centre of Gyration.*

EXAMPLE.

Required the radius of the circle of gyration in a water wheel, 30 feet diameter; the weight of the arms being 12 tons, shrouding 20 tons, and water 15 tons.

30 feet diameter, radius = 15 feet.

S	$20 \times 15^2 = 4500 \times 2 = 9000$	}	The opposite side of the water wheel must be taken.
A	$\frac{12 \times 15^2}{3} = 900 \times 2 = 1800$		

W	$15 \times 15^2 = 3375$	$= 3375$	
	$2 \times 20 + 12 = 64$	$\frac{14175}{79}$	
	W	15	
	79	79	$= 179$, the square root

of which is $13 \frac{4}{10}$ feet, the radius of the circle of gyration.

PUMPS.

There are two kinds of Pumps, Lifting and Forcing. The Lifting, or Common Pumps, are applied to wells, &c. where the depth does not exceed 32 feet; for beyond this depth they cannot act, because the height that water is forced up into a vacuum, by the pressure of the atmosphere, is about 34 feet.

The Force Pumps are those that are used on all other occasions, and can raise water to any required

L

height.—Bramah's celebrated Pump is one of this description, and shows the amazing power that can be produced by such application, and which arises from the fluid and non-compressible qualities of water.

The power required to raise water any height is equal to the quantity of water discharged in a given time, and the perpendicular height.

EXAMPLE.

Required the power necessary to discharge 175 Ale gallons of water per minute, from a pipe 252 feet high?

One Ale gallon of water weighs $10\frac{1}{4}$ lbs avoirdupois *nearly*.
 $175 \times 10\frac{1}{4} = 1799 \times 252 = \frac{453348}{44000} = 10.3$ horse power.

The following is a very simple Rule, and easily kept in remembrance.

Square the diameter of the pipe in inches, and the product will be the number of lbs of water avoirdupois contained in every yard length of the pipe. If the last figure of the product be cut off, or considered a decimal, the remaining figures will give the number of Ale gallons in each yard of pipe; and if the product contains only one figure, it will be tenths of an Ale gallon. The number of Ale gallons multiplied by 282, gives the cubic inches in each yard of pipe; and the contents of a pipe may be found by Proportion.

EXAMPLE.

What quantity of water will be discharged from a pipe 5 inches diameter, 252 feet perpendicular height, the water flowing at the rate of 210 feet per minute?

$$5^2 \times \frac{210}{3} = 175 \text{ Ale gallons per minute.}$$

$$5^2 \times \frac{252}{3} = 2100 \text{ lbs water in pipe.}$$

$\frac{2100 \times 210}{44000} = 10$ horse power required to pump that quantity of water.

The following Table gives the contents of a pipe one inch diameter, in weight and measure, which serves as a standard for pipes of other diameters, their contents being found by the following Rule.

Multiply the numbers in the following Table against any height, by the square of the diameter of the pipe, and the product will be the number of cubic inches avoirdupois ounces, and Wine gallons of water, that the given pipe will contain.

EXAMPLE.

How many Wine Gallons of water is contained in a pipe 6 inches diameter, and 60 feet long?

$$2.4480 \times 36 = 88.1280 \text{ Wine Gallons.}$$

In a Wine Gallon there are 231 cubic inches.

TABLE.

ONE INCH DIAMETER.			
Feet High.	Quantity in Cubic Inches.	Weight in Avoir. Oz.	Gallons Wine Measure.
1	9.42	5.46	.0407
2	18.85	10.92	.0816
3	28.27	16.38	.1224
4	37.70	21.85	.1632
5	47.12	27.31	.2040
6	56.55	32.77	.2448
7	65.97	38.23	.2856
8	75.40	43.69	.3264
9	84.82	49.16	.3671
10	94.25	54.62	.4080
20	188.49	109.24	.8160
30	282.74	163.86	1.2240
40	376.99	218.47	1.6300
50	471.24	273.09	2.0400
60	565.49	327.71	2.4480
70	659.73	382.33	2.8560
80	753.98	436.95	3.2640
90	848.23	491.57	3.6700
100	942.48	546.19	4.0800
200	1884.96	1092.38	8.1600

The resistance arising from the friction of water flowing through pipes, &c. is directly as the velocity of the water, and inversely as the circumference of the pipe.

The data given is a medium, and which is 1-5th of the whole resistance: this is the standard generally adopted, being considered as most correct.

EXAMPLE I.

What is the power requisite to overcome the resistance and friction of a column of water 4 inches diameter, 100 feet high, and flowing at the velocity of 300 feet per minute?

$$\frac{546.19 \times 4^2}{16} = 546.19, \text{ say } 546.2$$

$$\frac{546.2 \times 300}{44000} = 3.7, \text{ } \frac{1}{3}\text{th of which is } .7, \text{ therefore}$$

the power required to overcome the resistance occasioned by the weight and friction of the water will be $3.7 + .7 = 4.4$ H. P., say 4.5 horse power.

EXAMPLE II.

There is a cistern 20 feet square, and 10 feet deep, and placed on the top of a tower 60 feet high; what power is requisite to fill this cistern in 30 minutes, and what will be the diameter of the pump, when the length of stroke is 2 feet, and making 40 per minute?

$20 \times 20 \times 10 = 4000$ cubic contents of cistern.

$$\frac{4000}{30} = 133.3 \text{ cubic feet of water per min.}$$

$$\frac{133.3 \times 1000}{16} = 8331.25 \text{ lbs avoird. per minute.}$$

$$\frac{8331.25 \times 60}{44000} = 11.36 \text{ horse power, } \frac{1}{5}\text{th of which}$$

is $= 2.27 + 11.11 = 13.63$ horse power required.

$$2 \times 40 = 80 \quad \frac{133.3}{80} = 1.7 \times 144 = \frac{244.80}{.7854} = 311.7, \text{ now}$$

$\sqrt{311.7} = 17.6$ inches, diameter of pump required.

Founders generally prove the pipes they cast to stand a certain pressure, which is calculated by the weight of a perpendicular column of water, the area of which is equal to the area of the pipe, and the height equal to any given height.

To ascertain the exact pressure of water to which a pipe is subjected, a safety-valve is used, generally of 1 inch diameter, and loaded with a weight equal to the pressure required: for example, a pipe requires to stand a pressure of 300 feet, what weight will be required to load the safety-valve one inch diameter?

Feet.	Inches.	Ounces.
$300 \times 12 = 3600$	$\times .7854 =$	$\frac{2827.4400 \times 1000}{1728} = \frac{1636\frac{1}{2}}{16}$
$= 102 \text{ lbs } 4\frac{1}{2} \text{ oz. weight required.}$		

Each of the weights for the safety-valves of these Hydrostatic proving-machines are generally made equal to a pressure of a column of water 50 feet high, the area being the area of the valve.

50 feet of pressure on a valve 1 inch diam.	= 17.06 lbs
50 do. do. do. 1½ do.	= 26.65 do.
50 do. do. do. 1½ do.	= 38.38 do.
50 do. do. do. 2 do.	= 68.24 do.

In pumping, there is always a deficiency owing to the escape of water through the valves; to account for this loss, there is an allowance of 3 inches for each stroke of piston rod: for example, a 3 feet stroke may be calculated at 2 feet 9 inches.

There is a town, the inhabitants of which amount to 12000, and it is proposed to supply it with water, from a river running through the low grounds 250 perpendicular feet below the best situation from the reservoir.

It is required to know the power of an Engine capable of lifting a sufficient quantity of water, the daily supply being calculated at 10 Ale gallons to each individual: also, what size of pump and pipes are requisite for such?

12000 × 10 = 120000 gallons per day.

Engine is to work 12 hours, $\frac{120000}{12} = 10000$ gallons per hour.

$$\frac{10000}{60} = 166.6 \text{ gallons per minute.}$$

The pump to have an effective stroke of $3\frac{1}{2}$ feet, and making 30 strokes per minute.

$$\frac{166.6}{30} = 5.5533 \text{ gallons each stroke.}$$

$282 \times 5.6 = 1579.2$ cubic inches of water each stroke.

$\frac{1579.2}{45 \text{ in.}} = 35.1$ inches, area of pump.

$$\frac{35.1}{.7854} = 44.7, \text{ therefore } \sqrt{44.7} = 6.7 \text{ diam. of pump.}$$

The pipes will require to be at least the diameter of the pump; if they are a little more, the water will not require to flow so quickly through them, and thereby cause less friction.

The power of the Engine will be

$166.6 \text{ gall.} \times 10\frac{1}{2} \text{ lb} \times 250 \text{ feet} = 426925 \text{ momentum.}$

$$\frac{426925}{44000} = 9.7, \text{ add 1-5th} = 11.64 \text{ horse power.}$$

$$\frac{426925}{32000} = 13.3, \text{ ———} = 15.96. \text{ do. } \textit{Watt.}$$

$$\frac{426925}{27500} = 15.5, \text{ ———} = 18.6 \text{ do. } \textit{Desaguliers.}$$

$$\frac{426925}{22916} = 18.6, \text{ ———} = 22.32 \text{ do. } \textit{Smeaton.}$$

MISCELLANIES.

Required the weight of a cast iron ball of 3 inches diameter, supposing the weight of a cubic inch of the metal to be 0.258 lbs avoirdupois? HUTTON.

$$3^3 \times .5236 \times .258 = 3.6473976 \text{ lbs avoird.}$$

Required the weight of a hollow spherical iron shell, 5 inches in diameter, the thickness of the metal being 1 inch? HUTTON.

$$5^3 \times .5236 \times .258 = 16.88610 - 3.6473976 = 13.23871 \text{ lbs avoirdupois.}$$

It is proposed to determine the proportional quantities of matter in the earth and moon; the density of the former being to that of the latter, as 10 to 7, and their diameters, as 7930 to 2160. HUTTON.

$$\frac{7930^3 \times 10}{2160^3 \times 7} = 71 \text{ nearly; that is, 71 to 1 nearly.}$$

There are two bodies, of which the one contains 25 times the matter of the other, or is 25 times heavier; but the less moves with 1000 times the velocity of the greater; in what proportion then are the momenta, or forces with which they move? HUTTON.

$$\frac{1000}{25} = 40; \text{ that is, the less moves with a force 40 times greater.}$$

The following Table of the weight of cast iron pipes, gives the length of pipe according to the diameter of bore, as generally used in practice.

Diameter of bore in inches.
 Thickness of metal in inches.
 Length of pipe in feet.

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.										
			lb.	oz.	gr.				lb.	oz.	gr.				lb.	oz.	gr.								
1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2	3ft6	0	0	12	5 1/2	1	9	3	0	18	9	9	6	0	2	13	1	9	11	12					
		0	0	21	9			3	3	7			8	0	26			13 1/2	9	9	53	7			
		0	0	21	9			5	0	12			4	0	18			9	9	71	12				
	1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2	4ft6	0	1	4	6	1 1/2	9	2	0	0	9 1/2	9	5	1	0	14	1 1/2	9	83	16				
			0	1	8	9			2	2	21			6	1	6			9	9	113	24			
			0	2	0	9			3	1	17			0	2	20			14	9	60	4			
		1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2	6	0	1	16	6 1/2	2	9	4	0	16	10	9	4	1	10	14 1/2	2	9	72	16			
				0	2	10	9			5	2	20			9	5	1			26	9	91	0		
				0	3	10	9			2	0	16			9	6	2			14	9	121	14		
			1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2	6	0	2	20	6 3/4	3	9	3	3	20	10 1/2	9	4	2	14	15	3	9	60	24		
					0	1	12	9			4	1	21			9	5	3			7	9	92	2	
					0	1	3	6			9	6	0			14	9	7			0	9	123	6	
1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2				6	0	3	0	7	4	9	2	1	7	11	9	9	2	0	15	4	9	61	21		
					0	3	0	9			3	0	7			9	4	3			14	9	80	14	
					0	1	0	21			9	3	3			20	9	6			0	11	9	93	7
	1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2			6	0	1	14	7 1/2	5	9	4	3	5	11 1/2	9	7	1	7	15 1/2	5	9	130	26		
					0	2	0	8			9	6	2			4	9	9			3	20	9	163	5
					0	2	2	0			9	2	2			4	9	5			0	7	9	62	14
		1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2		6	0	1	10	7 3/4	6	9	3	1	6	12	9	6	1	12	16	6	9	81	14		
					0	1	3	12			9	4	0			22	9	7			2	8	9	100	10
					0	2	1	12			9	5	0			10	9	10			1	2	9	132	17
			1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2	6	0	3	21	8	7	9	7	0	0	12	9	5	0	24	17 1/2	7	9	171	6		
					0	2	0	4			9	4	1			25	9	6			2	8	9	70	22
					0	2	14	9			5	1	18			9	7	3			20	9	83	7	
1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2				6	0	3	0	8 1/2	8	9	5	7	1	12 1/2	9	10	3	0	21	8	9	101	20		
					0	3	0	21			9	7	1			16	9	5			1	16	9	140	8
					0	1	2	22			9	3	3			2	9	6			3	9	9	173	14
	1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2			6	0	2	1	10	9 1/2	9	4	2	26	13	9	8	1	0	22	9	9	213	4		
					0	2	3	17			9	5	2			22	9	11			0	21	9	293	21
					0	3	1	24			9	7	3			8	9	5			2	20	9	70	14
		1 1 1/2 2 2 1/2 3 3 1/2 4 4 1/2 5 5 1/2		6	0	1	3	9	10	9	4	0	0	13	9	7	0	14	23	9	9	52	20		
					0	2	2	0			9	9	9			9	9	8			2	7	9	70	14

The foregoing Table is found to be of great use in making out estimates of pipes:—for instance, it is required to know the weight of a range of pipes 225 feet long, $7\frac{1}{2}$ inches diameter of bore, and metal $\frac{3}{8}$ ths of an inch thick.

$$\begin{array}{r} 9)225 \\ \hline 25 \end{array} \text{ pipes in the whole length.}$$

One pipe weighs 4.0.22, which multiplied by 25, is equal to 104.3.18, or 5 tons, 4 cwt. 3 quarters, 18 lbs, weight of the whole range.

The following is a Table of the velocity of Motion, for boring cast iron cylinders, pumps, &c. and heavy Turning, with fixed cutters.

It will be observed, that the surface bored is constantly the same, 78.54 feet per minute; this velocity is found to be the most advantageous: a velocity greater than this, not only takes the temper out of the cutters, but also causing more heat, expands the metal; and if the machine stops but for a few seconds, a mark is left from the contraction of the metal.

Turning has a velocity double to that of boring.

TABLE.

BORING.		TURNING.	
Inches diameter	Revolutions of Bar $\frac{1}{2}$ Minute.	Inches diameter	Revolutions of Shaft $\frac{1}{2}$ Minute
1	25.	1	50.
2	12.5	2	25.
3	8.33	3	16.67
4	6.25	4	12.50
5	5.	5	10.
6	4.16	6	8.32
7	3.57	7	7.15
8	3.125	8	6.25
9	2.77	9	5.55
10	2.5	10	5.
15	1.66	15	3.33
20	1.25	20	2.50
25	1.	25	2.
30	0.833	30	1.667
35	0.714	35	1.430
40	0.625	40	1.250
45	0.56	45	1.12
50	0.5	50	1.
60	0.417	60	0.834
70	0.358	70	0.716
80	0.313	80	0.626
90	0.278	90	0.556
100	0.25	100	0.50

N. B. The progression of the cutters may be 1-16th of an inch for the first cut, and for the last 1-24th.

If hand tools are employed in turning, the velocity may be considerably increased.

It is proposed to divide the Beam of a Steel-yard, or to find the points of division where the weights of 1 2 3 4, &c. lbs on the one side, will just balance a constant weight of 95 lbs, at the distance of 2 inches on the other side of the fulcrum, the weight

of the Beam being 10 lbs, and its whole length 36 inches.

$$36 : 10 :: 2 : 55 = \frac{20.000}{36} = .55 \text{ weight of short arm.}$$

2 = length of short arm.

36 — 2 = 34 = length of long arm.

10 — .55 = 9.45 = weight of long arm.

95 × 2 = 190 momentum of weight at end of short arm.

.55 × 1 = .55 do. of short arm.

190.55 whole momentum of weight and arm.

$$\frac{9.45 \times 34}{2} = 160.65 \text{ momentum of long arm.}$$

190.55 — 160.65 = 29.90, or 30, the excess of momentum.

$$\frac{30}{1} = 30 \text{ inches from the fulcrum for the one lib weight.}$$

$$\frac{30}{2} = 15 \text{ two do.}$$

$$\frac{30}{3} = 10 \text{ three do.}$$

$$\frac{30}{4} = 7\frac{1}{2} \text{ four do.}$$

Niven, Printer, Prince's Street, Glasgow.



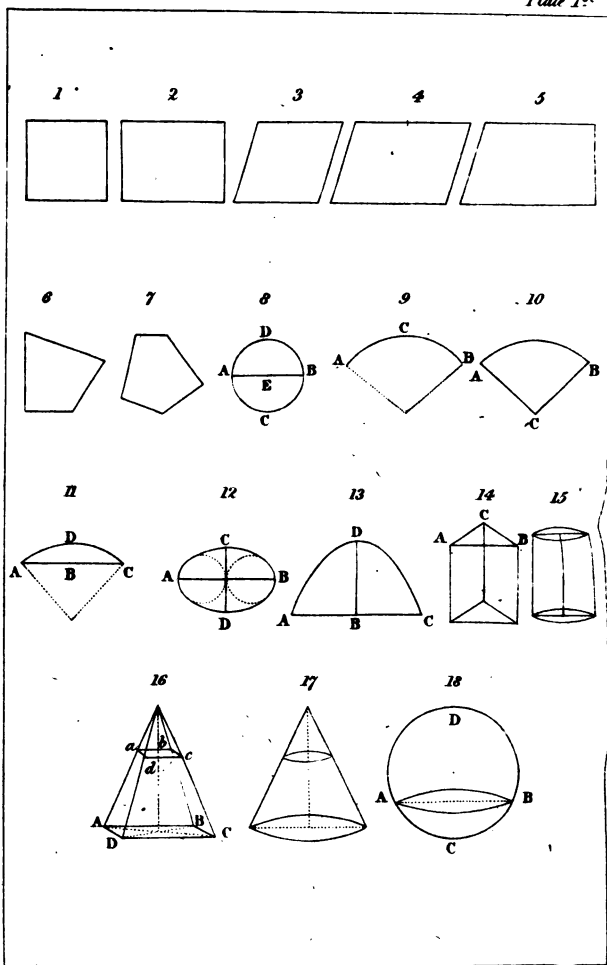
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MENSURATION.

Plate 1st



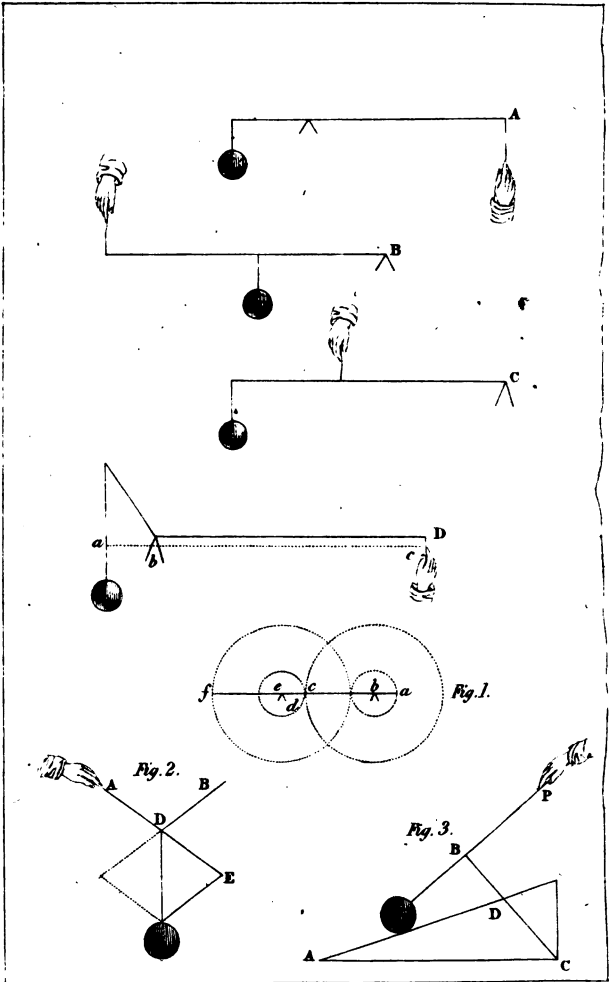
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EXPLANATION OF PLATE I.

- No. 1. A Square.
2. A Rectangle.
3. A Rhombus.
4. A Rhomboid.
5. A Trapezoid.
6. A Trapezium.
7. An Irregular Polygon.
8. A Circle, AE the radius, AEB the diameter, $ACBD$ the circumference.
9. An Arc of a Circle, ACB .
10. A Sector of a Circle, ACB .
11. A Segment of a Circle, $ABCD$.
12. An Ellipsis or Oval, AB the long diameter, CD the short diameter.
13. A Parabola, ABC the base, BD the perpendicular height.
14. A Prism, ABC the perimeter; or the circumference of the end of a cylinder, is the perimeter of that cylinder.
15. A Cylinder.
16. A Pyramid, $ABCD$ the base; $ABCabcd$ the frustum.
17. A Cone.
18. A Sphere, $ABCD$ the circumference, ABC a segment.

MECHANICAL POWERS.

Plate 2nd



—Sven—

EXPLANATION OF PLATE II.

A—Lever of the first order.

B—Lever of the second order.

C—Lever of the third order.

D—Bended Lever: the effective power and weight on a bended lever, is as the distance between the points of action and the fulcrum, as $a b c$ b . The distance being taken at right angles to the direction of the forces.

Fig: 1. A Diagram, explanatory of the Wheel and Axle.

— 2. A do. explanatory of the Pulley, when the directions of the cords are not parallel.

— 3. A Diagram, explanatory of the inclined plane, when the power is not in a direction with the plane.

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