PRINCIPLES OF CONSTRUCTION

AND

EFFICIENCY OF WATER-WHEELS.

BY

WILLIAM DONALDSON, M.A.,
ASSOC. INST. C.E.,

AUTHOR OF "A TREATISE ON OBLIQUE ARCHES," "SWITCHES AND CROSSINGS," "TABLES FOR PLATELAYERS," AND "NEW FORMULAS FOR THE LOADS AND DEFLECTIONS OF SOLID BEAMS."

LONDON:
E. & F. N. SPON, 48, CHARING CROSS.
NEW YORK: 46, BROOME STREET.
1876.

LONDON:
PRINTED BY WILLIAM CLOWES AND SONS,
STAMFORD STREET AND CHARING CROSS.

PRINCIPLES OF CONSTRUCTION

ERRATA.

Page 89. "Initial pressure" *should be* "nett initial pressure."

„ 91. "$\sqrt{2gH}$, is ·53" *ought to be* "$\sqrt{2gH}$, is $= \frac{565}{870} = \cdot 65$."

„ 92. *For* the sentence beginning, "The corresponding value," and ending "5,100 cubic feet," *substitute* "If $c = \cdot 84$, the theoretical discharge would be 5,100 cubic feet, and the theoretical value of the ratio of best speed would be $= \frac{\cdot 65 \times \sqrt{\cdot 84}}{\sqrt{\cdot 74}} = \cdot 69$."

any interval by the height through which it falls, the units of time, weight, and space generally adopted being a second, a pound, and a foot respectively. Professor Rankine in his work on "Prime Movers" uses the word *energy* to express the same thing. Neither according to its strictly etymological, nor any acknowledged derivative meaning, can the word energy be rightly used to express the idea. It is more nearly synonymous with efficiency. Perhaps the difference between the two, and the reasons for preferring the word "power" cannot be better illustrated than by

PRINCIPLES OF CONSTRUCTION

AND

EFFICIENCY OF WATER-WHEELS.

INTRODUCTION.

The use of water as a motive power has been so well known from the earliest ages of which we have any record, and the best mode of its application has formed the subject of so many scientific treatises, that the Author feels he cannot advance any new views, or establish any new rules of construction of the more ancient forms of wheels. Modern inventions for utilising the impulsive power of water, such as turbines, do not seem, however, to have been thoroughly investigated; at least, the Author has not been able to meet with any treatise, in which detailed rules of construction have been given; or if detailed rules of construction have been given, no scientific reason has been laid down for their adoption.

Part I. is devoted to the investigation of the effect of the impulse of water against vanes, the general principles of construction, and the efficiency of the different classes of vertical wheels.

Part II. treats of the efficiency, principles, and details of construction of the working parts of the three classes of turbines—outward, inward, and parallel flow.

Smeaton and the older writers on this subject use the word *power* to express the product of the weight of water falling during any interval by the height through which it falls, the units of time, weight, and space generally adopted being a second, a pound, and a foot respectively. Professor Rankine in his work on "Prime Movers" uses the word *energy* to express the same thing. Neither according to its strictly etymological, nor any acknowledged derivative meaning, can the word energy be rightly used to express the idea. It is more nearly synonymous with efficiency. Perhaps the difference between the two, and the reasons for preferring the word "power" cannot be better illustrated than by saying, you may be powerful without being energetic, and energetic without being powerful.

To utilise this power, water-wheels of different kinds have been invented. They may all, however, be divided into two classes.

(1.) Those which are driven partly by the statical weight of the water acting through a portion of the whole fall, partly by the momentum acquired by the water before it strikes the wheel.

(2.) Those which are driven by momentum acquired by the water only.

Wheels of the first class are designated "weight and impulse," those of the second "impulse" wheels.

The power which the wheel is capable of transmitting to the machinery is called by Smeaton the effective power of the wheel. This, owing to the friction of the bearings and other resistances, and the impossibility of utilising the whole power of the fall, whether it acts partly by weight, partly by impulse, or wholly by impulse, is always less than the gross power of the fall. The ratio, effective power ÷ gross power of fall, is called the co-efficient of efficiency of the wheel.

The efficiency can only be determined by actual experiment. Co-efficients determined by calculations, based upon assumptions which, although more nearly true in some cases than in others, never exactly represent the actual state of the case, are never sufficiently near the truth to determine the absolute power of any wheel, even when this is considered apart from the efficiency lost by friction of the bearing surfaces or by contact with external resistances, such as the tail water in a vertical wheel race, or the water surrounding a submerged horizontal wheel.

Such co-efficients will, however, enable us to compare with sufficient exactness the relative efficiency of different wheels, and of the same wheel under different circumstances, and the theoretical investigation into their value will show us how to determine the form of the wheel of each class, which will, in practice, develop the highest efficiency.

The co-efficient of efficiency of turbines is usually stated to be equal to that of high breast wheels, viz., about ·75. The investigations of the Author have led him to the conclusion that this efficiency cannot much exceed ·5. The argument of those who uphold the higher co-efficient is simply this. The power lost by the water must have been communicated by the machine; if not, what has become of it? If such an argument be applicable to the case of one hydraulic machine, it must be applicable to all. Thus the water in the tail race of an undershot wheel with flat vanes moving at its best velocity, moves with only half the original velocity. Seventy-five per cent. of the original power of the current has been lost, therefore ·75 is the co-efficient of efficiency of undershot wheels.

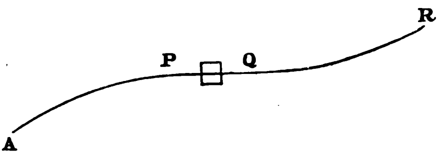
PART I.

IMPULSIVE ACTION OF WATER AND VERTICAL WATER-WHEELS.

The simplest case of the impulsive action of water is that of a flat vane immersed in a stream at right angles to its direction. The following investigation of the effective power exerted by the stream on such a vane is given in Mosely's "Hydrostatics."

Let the curve A P Q R represent the direction of the motion of the stream between the points A and R. Let us consider the

FIG. 1.



motion of a small cylindrical element of the fluid P Q. Let K be the area of either end of the cylinder, S the distance of P from A and P Q $= \delta s$. Since the pressure may vary from A to R, but is always of the same value at the same distance from A, if p be the pressure at P, the pressure at Q will be $p + \delta p$. Let S be the accelerating force on the end P, ρ the density of the liquid, then the moving force on the element P Q will manifestly be

$$K \rho \delta s S - K \delta p;$$

if, therefore, v be the velocity of the point P, the equation of motion of the point P, when δs is indefinitely diminished, will be

$$v \frac{dv}{ds} = S - \frac{1}{\rho} \frac{dp}{ds},$$

whence

$$\tfrac{1}{2} v^2 = \int S \, ds - \frac{p}{\rho} \quad (A),$$

we may apply equation (A) in the two following ways for determining the moving force of the water on the vane.

(1.) Let the vane be stationary.

Let p' be the pressure in front of the vane after immersion;

since the motion of the water in direction A P Q R is wholly destroyed, we have

$$o = \int S\,d s - \frac{p'}{\rho},$$

and therefore K $(p' - p)$ the moving force exerted on the vane is equal to

$$\tfrac{1}{2} K \rho v^2 = K g \rho . \frac{v^2}{2 g},$$

or the moving force is equal to the weight of a column of water whose height is equal to that due to the velocity of the current and base equal to the area immersed.

(2.) Let the immersed vane have a velocity *u*.

The effort of the stream on the vane may be viewed in two lights.

(*a*.) Since the moving force on the vane when stationary is equal to the weight of a column of water whose height is equal to that due to the velocity of the current, we might infer that when the vane is in motion the moving force would be equal to the weight of a column of water whose height is equal to that due to the difference between the velocities of the water and the vane, or to

$$K g \rho \frac{(v - u)^2}{2 g},$$

in which case effective power exerted per second would be

$$K g \rho . \frac{(v - u)^2 u}{2 g},$$

from which we get these theoretical values of the ratios.

$$\frac{\text{Best velocity of vane}}{\text{Velocity of current}} = \frac{1}{3} = \cdot 34$$

$$\frac{\text{Moving force at best velocity}}{\text{Moving force when stationary}} = \frac{4}{9} = \cdot 45$$

$$\frac{\text{Effective power}}{\text{Gross power}} = \frac{4}{27} = \cdot 15$$

$$\frac{\text{Effective power}}{\text{Power lost}} = \frac{1}{6} = \cdot 17$$

the gross power being equal to the product of the weight of water passing in one second, viz.: $K g \rho v$ multiplied by the height due to the velocity of the stream, and the power lost to the product of the same weight of water multiplied by the difference between the heights due to the initial and final velocity, viz.: $\frac{8v^2}{9g}$.

(*b*.) Referring to equation (A), if p' represent the pressure, when the vane is immersed on the front of the vane, since the velocity in the direction A P R must be equal to that of the vane, we have

$$\tfrac{1}{2}u^2 = \int S\,ds - \frac{p'}{\rho},$$

and therefore the moving force on the vane will be equal to

$$K g \rho . \frac{(v^2 - u^2)}{2g},$$

and the effective work done per second

$$\frac{K g \rho (v^2 - u^2) u}{2g},$$

whence

$$\frac{\text{Best velocity of vane}}{\text{Velocity of current}} = \sqrt{\frac{1}{3}} = \cdot 58$$

$$\frac{\text{Moving force at best velocity}}{\text{Moving force when stationary}} = \frac{2}{3} = \cdot 67$$

$$\frac{\text{Effective power}}{\text{Gross power}} = \frac{2}{3\sqrt{3}} = \cdot 385$$

$$\frac{\text{Effective power}}{\text{Power lost}} = \sqrt{\frac{1}{3}} = \cdot 58$$

In the above investigations, it has been assumed that the pressure in rear of the vane is equal to the pressure of the fluid before the immersion of the vane. Since the pressure before immersion exceeds very little that of the atmosphere, because the depth of immersion is small, this assumption must be looked upon as practically correct.

The solution given in (*b*) is rigorously exact on these data. That given in (*a*) is a mere deduction from the calculated moving force on the vane when stationary, and has only been given because the theoretical co-efficients obtained by that mode of solution are the same as those adopted by Smeaton in comparing the results of his experiments with those of theoretical investigation.

The following table gives values selected from the results of twenty-seven experiments made by Smeaton to determine the effective power of a current on a sunk vane, the exact velocity of impact and the exact quantity of water expended per second having been ascertained with the greatest nicety. They are taken from a Paper read before the Royal Society in 1759. A description of the mode of performing the experiments is given in an appendix, together with a tabular statement and analysis of the results:—

Height due to velocity of Impact.	Velocity of Vane. / Velocity of Water.	Moving force at best velocity. / Moving force stationary.	Effective power. / Power lost.	Effective power. / Gross power.	Efficiency of Wheel.	Efficiency of Wheel Actual. / Efficiency of Vane Theoretical.	Remarks.
″							
15	·34	·73	·48	·33	·30	·77	Small depth of current.
5	·52	·83	·42	·30	·27	·70	Great depth of current.
4	·38	·70	·40	·28	·25	·64	Small depth of current.
	·58	·67	·58	·39	..	..	Theoretical values.

When a jet strikes a flat vane, which is moving freely in the air unimpeded by tail water, the theoretical effective power exerted may be accurately calculated by a direct application of the three laws of motion, which may be thus enunciated:—

1. A particle, if at rest, will continue at rest, and if in motion will move in a straight line with uniform velocity, unless it is acted on by an extraneous force.
2. When a particle is in motion under the action of any force, the acceleration of the particle estimated in any assigned direction is wholly due to the force resolved in that direction, and is the same in intensity as if that force alone acted on the particle at rest.
3. When a force or pressure acts on a particle, the moving force on the particle is proportional to the force or pressure acting on it.

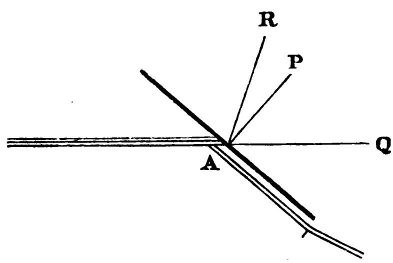
In applying these laws to ascertain the moving force exerted by the water against the vane, we must go through the inverse process, and ascertain how much the jet is deflected from its course by contact with the vane. The moving force necessary to do this, the value of which those laws enable us to calculate, is evidently equal to the moving force exerted by the jet on the vane, since action and reaction are equal and opposite.

The direction of motion of a jet, at the instant it strikes a vane, may be either at right angles to the plane of contact, or inclined to that plane at any angle between zero and 90°. If we leave out of consideration the effect of friction between the fluid and the surface of the vane, it is evident that that component alone of the moving force of a jet which acts at right angles to the plane of contact can have any effect on the motion of the vane, so that if it were free at the instant the impulse takes place the vane would begin to move in a direction at right angles to the plane of initial contact.

If the vane be not capable of moving freely, but is constrained by some extraneous force, such as a rigid connection with other parts of a machine, the absolute direction of motion of the vane at the instant the jet strikes may be in a direction inclined at an angle to the normal to the plane of initial contact. In this case the moving force exerted by the jet on the vane will be equal to the component of the whole moving force resolved at right angles to the initial plane of contact, and the other component parallel to this plane will be due to the action of the above-mentioned extraneous force.

Hence it follows, that in order to ascertain the moving force exerted by a jet on a plane vane, it will simply be necessary to resolve the velocity of the jet in two directions, one parallel to the vane—which, leaving friction out of consideration, will not be affected by the impulse—and the other at right angles to the vane. The moving force exerted on the vane will be equal to the momentum lost by this component.

FIG. 2.



Let A Q be the direction of motion of the jet, A R that of the vane. At the point A, where the jet strikes the vane, draw A P at right angles to the vane.

If then

$\angle$ P A Q $= a$
$\angle$ P A R $= \delta$
velocity of jet in feet per second $= v$
" vane " $= u$

we shall have

Velocity of water parallel to vane $= v \sin a$
" " at right angles $= v$ cosin a
Tangential velocity of vane $= u \sin \delta$
Normal velocity of vane $= u \cos \delta$.

And if P represent the whole moving force in direction A R of the motion of the vane due to the impulse of the water and the other extraneous force, the component of the moving force due to the impulse of the water will be equal to P $\cos \delta$, and this will be equal to the momentum lost at right angles to the vane, or—

$$P \cos \delta = \frac{W (v_1 \cos a - u \cos \delta)}{g}$$

where W is the weight of water discharged in a second; and therefore the effective work done will be given by the equation

$$P u \cos {}^2\delta = \frac{W (v_1 \cos a - u \cos \delta) u \cos \delta}{g},$$

which is maximum when $u \cos \delta = \frac{v_1 \cos a}{2}$ and is equal to $\frac{W v_1^2 \cos {}^2a}{4 g}$. Since the power of the jet is equal to $\frac{W v_1^2}{2 g}$, we have efficiency $= \frac{\cos {}^2a}{2}$.

If v_2 be the final absolute velocity of the jet

$$v_2^2 = v_1^2 \sin {}^2a + u^2 \cos {}^2\delta;$$

or for speed of best efficiency

$$v_2^2 = v_1^2 \sin {}^2a + \frac{v_1^2 \cos {}^2a}{4}$$
$$= v_1^2 - \frac{3 v_1^2 \cos {}^2a}{4}.$$

Since the power lost by the jet is given by the equation

$$\frac{W (v_1^2 - v_2^2)}{2 g} = \frac{3 W v_1^2 \cos {}^2a}{8 g},$$

the effective power is equal to two-thirds of the power lost—the proportion shown to exist in the case of a flat vane immersed in a stream being about three-fifths.

The solutions which Professor Rankine gives in his work on "Prime Movers" are based on three assumptions diametrically opposed in principle to those here enunciated:

(1.) It is assumed that in whatever direction the vane may move, its velocity in that direction is wholly due to the impulse of the water.

(2.) It is assumed that the relative velocity after impact is equal to the relative velocity before impact, if friction be left out of consideration.

(3.) It is assumed that the relative velocity of the jet, after striking a curved vane, is unaffected by the curvature of the vane.

Following out his investigations based on these assumptions, he arrives at the resulting final equation:

$$P u = \frac{W (v_1^2 - v_2^2)}{2g},$$

where P, u, are the moving force and velocity in any assigned direction. Upon this equation he puts the following interpretation: "The energy exerted by the water on the vane is equal to the actual energy lost by the water—a consequence of the assumption that friction is insensible." If this were true it would follow that all machines would be equally efficient which made the final velocity the same.

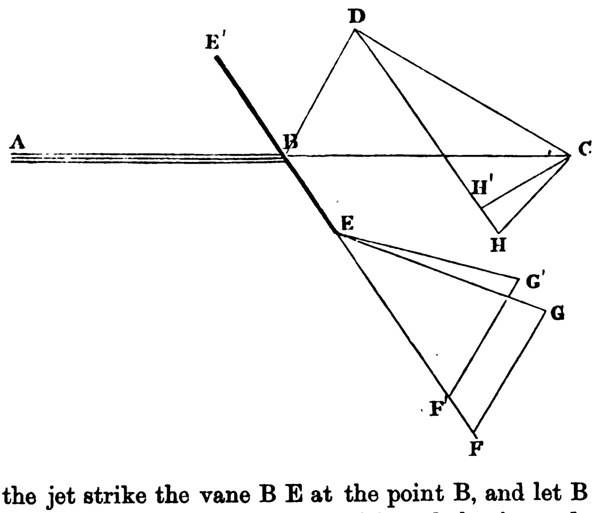
A free vane acted on by a jet of water will move in the direction of the resultant of the forces exerted on it. The moving force in the direction of motion will not be equal to the sum of the separate moving forces at each point, but to the sum of their components in that direction. Since general solutions contain all particular solutions, his general solution must contain that of a flat vane. It does so only when $\delta = o$, because then the magnitude of the relative velocity after impact does not affect the problem. Professor Rankine has preferred to take this case separately, and finally arrives at the value $\frac{\cos^2 a}{2}$ for the efficiency, although in the intermediate step the error involved in assumption No. 1 is introduced. Thus whilst $u \cos \delta$ is given as the value of the normal velocity, the moving force producing this velocity is put equal to P sec. δ, or the less velocity is produced by the greater moving force, which is manifestly absurd.

The Author has assumed that the jet is composed of a number of inelastic molecules of indefinitely small mass $\delta\, m$. If the momentum $\delta\, \mu .\, v_1$ be resolved tangentially and normally to the plane of impact, the magnitude of the first component will remain unaltered, whilst that of the second will become $\delta\, m.\, u \cos \delta$. Therefore the impulse communicated by one molecule will be equal to $\delta\, m\, (v_1 \cos a - u \cos \delta)$ and the whole impulse to $\frac{W\, (v_1 \cos a - u \cos \delta)}{g}$.

As regards assumption No. 1, it is manifest that the motion of the vane, if free, whether curved or plane, could only occur in one definite direction under the action of a jet, and that if the vane move in any other there must be some force other than that of the jet constraining it to move in that direction, to which part of the velocity in that direction is therefore owing.

That assumption No. 2 is erroneous may be demonstrated by applying the principles of solution laid down by Rankine for the general case to the case of a flat vane struck obliquely by a jet.

Fig 3.



Let the jet strike the vane B E at the point B, and let B C, B D represent respectively the initial velocities of the jet and vane in magnitude and direction. Join D C.

D C will represent in magnitude and direction the motion of the jet relatively to the vane before impact.

Draw E F parallel to the face of the vane and equal to D C; also through F draw F G parallel and equal to B D, and join E G. Through D draw D H parallel and equal to E F, and join H C. If, therefore, the relative velocity of the water and the vane after impact be equal to the relative velocity before impact, E G will represent in magnitude and direction the final absolute velocity of the water, and H C the direction and magnitude of the change of motion of the jet during the interval of contact with the vane. Also D H is equal to D C, and the two angles D H C, D C H are equal to each other, therefore each must be less than a right angle, and therefore the direction C H, in which the jet has been deflected, is not at right angles to D H, which is parallel to the surface of the vane.

Therefore the change of velocity caused by impulse against a plane vane has been in a direction other than normal to the vane, friction being left out of consideration, which is impossible. Therefore the relative velocity after impact cannot be equal to the relative velocity before impact. Draw C H′ at right angles to D H. In like manner it may be shown that no other line than C H′ can represent in magnitude and direction the change of motion of the jet, therefore the relative velocity after impact will be represented by the line D H′. In E F take the point F′, such that E F′ = D H′ and draw F′ G′ parallel and equal to B D. Join E G′. Then will E G′ represent in magnitude and direction the absolute velocity of the jet after leaving the vane.

It follows from this that when a jet strikes a flat vane normally, which is likewise moving freely in a normal direction, no change in direction of the motion of jet, but in magnitude only, is occasioned by the impact, so that the final absolute velocity is simply equal to that of the vane, and the relative velocity to the difference between the velocity of the jet before impact and that of the vane. The lateral motion which is observed when a jet strikes a flat vane normally is owing to the pressure exerted by the succeeding particles upon those preceding them, which causes the last to move laterally out of the way. This action and reaction between the particles of water themselves causes the jet to increase perceptibly before it reaches the vane, so that the efficiency, ascertained experimentally, will be less than that given by theory, since the velocity with which each particle strikes the vane will be less than the velocity of the jet.

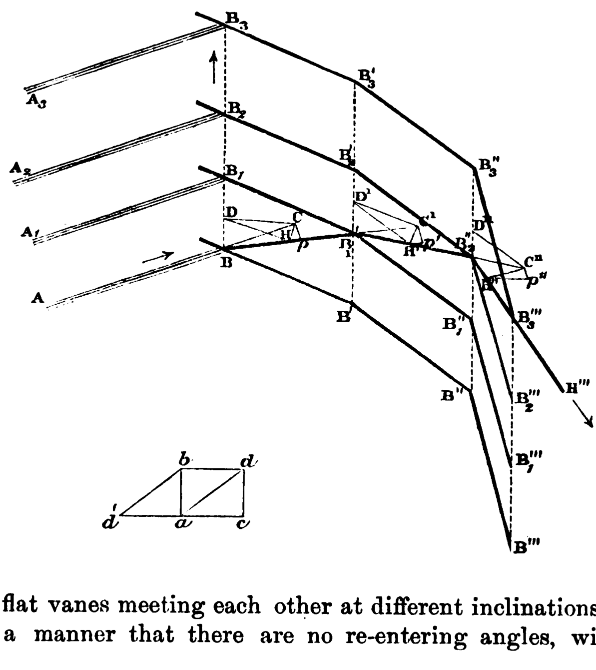
Hence we see that the efficiency of a cup vane, whether segmental or hemispherical, will be simply equal to that of a flat vane moving with its plane at right angles to the jet, if the jet strikes the cup vane normally.

If a jet strike a curved vane in a direction not normal to the tangent plane at the point of impact, the velocity with which the jet glances off from the curved vane will be the same as that with which it would glance off from a flat vane tangential to the surface at the point of impact if this velocity were not diminished by the curvature of the surface intervening between the point where the jet strikes and leaves the vane.

Professor Rankine affirms that this curvature, if gradual, will have no effect on the relative velocity, "because the deflecting force acting on the particle of water is wholly normal to the direction of motion." Experiments made, however, to ascertain the resistance to the flow of water in pipes, show that bends, however easy, do present a resistance to the flow in addition to the surface friction, which cannot be greater round a curve than along a straight line. In fact, the statement that the deflecting force acts always normally to the direction of motion in the same manner as a constant central force on a particle moving in a circular orbit is not correct. In the case of curved vanes, the problem to be solved, in order to ascertain the effective power of the jet, may be illustrated by supposing the curved vane to be the limiting position of a number of flat vanes meeting each other at various angles, when their number is indefinitely increased and their length diminished indefinitely.

The following graphic solution of the case of a vane composed of

FIG. 4.



three flat vanes meeting each other at different inclinations, but in such a manner that there are no re-entering angles, will fully illustrate the nature of the problem, and show how to determine the form best suited to develop the greatest effective power. For simplicity's sake, we will suppose that the motion of the vane is linear, and that gravity does not act on the water after impact with the first vane, so far as change of velocity is concerned.

Let the compound vane B B′ B″ B‴ be struck by a jet of water A B, the direction and magnitude of the velocity of which is represented by the line B C.

Let B D represent the velocity of the vane in direction and magnitude due to the effective power exerted by the water and some other constraining force.

Join D C and draw D H parallel and C H perpendicular to B B′, then will C H represent in magnitude and direction the space through which the water is deflected per second by impulse against the chord B B′, and B H will represent in magnitude and direction the absolute velocity of the jet after impact on the first vane.

Draw B′ B_1' parallel to the direction of motion meeting B H produced in B_1'. Draw B_1' B_1 and B_1' B_1'' parallel and equal respectively to B′ B and B′ B″, and through B_1'' draw B_1'' B_1''' parallel and equal to B″ B‴, then will B_1 B_1' B_1'' B_1''' represent the position of the vane at the instant the jet strikes the chord B′ B″ represented by B_1' B_1''.

Produce B B_1' to C′, and make B_1' C′ equal B H the absolute velocity after impact on the first chord. Draw B_1' D′ equal to B D and parallel to the direction of motion, then will D′ C′ represent in magnitude and direction the relative velocity of the water before impact on the chord B_1' B_1''.

Draw C′ H′, D′ H′ perpendicular and parallel respectively to B_1' B_1'', then will C′ H′ represent the space through which the water has been deflected by impulse against the chord B_1' B_1'' and B_1' H′ will represent in direction and magnitude its absolute velocity.

Through B_1'' draw B_1'' B_2'' parallel to the direction of motion meeting B_1' H′ produced in B_2''. Draw B_2'' B_2''' and B_2'' B_2' parallel and equal to B″ B‴ and B″ B′ respectively, and B_2' B_2 parallel and equal to B′ B, then will B_2 B_2' B_2'' B_2''' represent the position of the vane at the instant the jet strikes the chord B_2'' B_2'''.

By a similar construction it may be shown that C″ H″ represents in magnitude and direction the space through which the water has been deflected in a second by impact with the chord B_2'' B_2''' and B_2'' H″, the absolute velocity. Also that B_3 B_3' B_3'' B_3''' represents the position of the vane at the instant the jet leaves it. It is plain also that $B\,B_1{}'\,B_2{}''\,B_3{}'''$ will represent the absolute path of the water during contact with the vanes.

In the above investigation the jet A B is supposed to move parallel to itself into the positions $A_1\,B_1$, $A_2\,B_2$, &c., so as always to strike the first of the three chords of the vane.

Through C C′ C″ draw $C\,p$, $C'\,p'$, $C''\,p''$ parallel to the direction of motion, and through H H′ H″ draw $H\,p$, $H'\,p'$, $H''p''$ at right angles to the direction of motion meeting $C\,p$, $C'\,p'$, $C''\,p''$ in p, p', p'' respectively, then will the sum of the three last represent the space through which the water has been deflected per second parallel to the direction of motion, and of the three first that at right angles to it.

Take $a\,b$ equal to the sum of the three first, and $a\,c$ at right angles to it equal to the sum of the three last. Construct the parallelogram $a\,b\,d\,c$, and join $a\,d$. $d\,a$ will represent in magnitude and direction the space through which the water has been deflected by the vane per second, and this multiplied by the mass of the water discharged per second will be equal to the resultant moving force exerted per second; also $a\,b$ and $b\,d$ multiplied by the same mass of water will represent the moving force in direction of motion and the strain on the machinery perpendicular to the direction of motion respectively. If therefore W be the weight of water discharged per second, we shall have

$$\text{Strain on machinery} \quad . \quad . \quad . \qquad = W\frac{b\,d}{g}$$

$$\text{Moving force in direction of motion} = \frac{W\,a\,b}{g}$$

$$\text{Effective power exerted} \quad . \quad . \quad . \quad = \frac{W\cdot a\,b\cdot B\,D}{g}$$

$$\text{Gross power of jet} \; . \quad . \quad . \quad . \quad . \quad = \frac{W\cdot B\,C^2}{2\,g}$$

$$\frac{\text{Effective power}}{\text{Gross power}} = \frac{2\,a\,b\cdot B\,D}{B\,C^2} \quad . \quad . \quad = \quad \cdot 42$$

$$\frac{\text{Effective power}}{\text{Power lost}} = \frac{2\,a\,b\cdot B\,D}{B\,C^2 - B_2{}''\,H''^2} \quad = \quad \cdot 50$$

Produce $B_2{}''\,B_3{}'''$ to H‴, making $B_3{}'''\,H'''$ equal to $B_2{}''\,H''$, then will $B_3{}'''\,H'''$ represent in magnitude and direction the final absolute velocity. It is plain therefore that the addition of more chords would have increased the effective work of the vane. If one other chord only be added parallel to the direction of motion, since the change in direction of the absolute velocity would be wholly at right angles to the direction of motion, such an additional chord would produce no additional efficiency. Also the nearer the additional chord is to such a chord the less will be the effective power obtained from the gross power lost, and the less the final absolute velocity. On the contrary, the more remote the additional chord is from the limiting position of parallelism to the direction of motion, the greater will be the ratio of the effective force to the power lost, and the greater the final absolute velocity. It therefore follows, that to utilise to the utmost the final absolute velocity $B_3''' H'''$, there ought to be a succession of chords between $B_2'' B_3'''$, and the limiting position $B_3''' B'''$ meeting each other at indefinitely small equal angles. If these conditions were observed, the change in direction of motion of the absolute path would be gradual and uniform, and therefore the absolute path itself circular, since the same reasoning applies to every successive change of direction between the different chords. Therefore, in order to develop the maximum effective power obtainable from a curvilinear vane, the relative path—that is, the shape of the vane—must be so designed that the absolute path of the jet during contact will be circular. Also, leaving friction out of consideration, the easier the radius the greater will be the efficiency, because the more numerous and minute will be the successive changes of direction.

It is plain that any effective power developed after the direction of the absolute velocity has once become parallel to the direction of motion will retard the motion, therefore the limiting position of the last tangent to the curve of each of the paths will be one of parallelism to the direction of motion. In order to comply with the condition of gradual change of direction, the first tangent to the relative path ought to be parallel to the initial relative motion of vane and water.

If the motion of the vane be curvilinear, as is the case in all water-wheels, the velocity per second of different parts of the vane will vary. In the case of wheels of large diameter the linear velocities of different parts of the vane will be practically the same; but in the case of wheels of small diameter, such as turbines, the linear velocities of the inner and outer edges of the vane will vary very materially. The principles of solution are, however, the same in both cases, and in both therefore the absolute path ought to be circular.

The determination of the elements of this absolute path, and of the form of the vane necessary to cause the water to describe this path constitutes the whole secret of the art of constructing turbines. The method of doing this will be fully discussed in Part II.

In the case of vertical water-wheels, in which the water glances off at the same edge at which it strikes, the effective theoretical power, as the Author will endeavour to prove in discussing the theory of the Poncelet wheel, cannot much exceed that obtained from a flat radial vane. It remains then only to investigate the nature of the action of the water during that part of the fall in which it acts by its weight only, to be in a position to discuss the principles of construction and efficiency of these wheels.

If a support, to which a heavy body is attached, be moving vertically downwards with a uniform velocity less than that due to the height through which the body may have fallen before it reached the support, the downward force exerted by that body on the support after impact will be the same as if that body and the support were at rest.

If the support have a downward acceleration of motion per second, which must necessarily be less than the acceleration due to gravity, the pressure on the support will be less than when it is at rest or moving with a uniform velocity by the quantity $\frac{W \delta v}{g}$, W being the weight of the body, and δv the acceleration per second. On the contrary, if the support have a retardation of δv feet per second, the pressure on the support will be increased by $\frac{W \delta v}{g}$. Now, in the case of a vertical water-wheel moving with a uniform angular velocity, the vertical velocities of points in the periphery near the top and bottom will be least; but the increments on leaving and the decrements on approaching either of these points in vertical velocity per second will be greatest. Near the middle part of the fall the vertical velocities of the periphery will be greatest, but the increments and decrements least.

It follows, then, that the effective power of the water developed in falling a given vertical height near the top of a wheel is less than that developed in falling through the same height near the middle, whilst the effective power developed in falling through the same height near the bottom is greater. If, then, by means of a breast or other contrivance the water during that part of the descent which lies below the centre of the wheel can be so retained in contact with the buckets as to insure the equality in value of the tangential components of the weight at all points equidistant from a vertical line through the axis, it follows, leaving friction against the breast out of consideration, that if the point at which the water enters the wheel is nearer to the summit than the point of discharge is to the bottom, the effective power developed will be less, if at the same distance equal, and if more remote greater, than that due to the height through which the water has fallen in contact with the wheel. At first sight it would oppear that this last conclusion could not possibly be true. Each of the three, however, admits of a very simple explanation. In the first case, the whole of the excess of the increment of the vertical velocity gained in the upper quadrant over that lost in the lower quadrant is not utilised. In the second case, it is exactly utilised. In the third case, not only is the whole increment of vertical velocity gained in the upper quadrant utilised in the lower, but part also of the initial vertical velocity of impact is destroyed.

It appears, then, that the water in the buckets in the lower quadrant acts partly by weight, partly by momentum, or in other words, a low breast wheel moving at the same rate as a high breast wheel relatively to the initial velocity of impulse of the water, utilises more of the head due to that velocity, and is so far more efficient.

The increments and decrements in vertical velocity per second due to the angular motion of the wheel are, however, so small compared with the acceleration per second due to gravity, that we may assume the effective power of that part of the fall during which the water acts by its weight only as equal to the gross power.

Although a given weight of water near the middle will counterbalance a greater weight hung round the axis than the same weight nearer the vertical through the axis, the work done in both cases in falling through the same vertical height will be the same, because the respective angular motions will be inversely as the respective momenta.

The preceding investigations enable us to determine the best proportions and theoretical efficiency of all kinds of vertical wheels in which the water glances off the same edge at which it strikes the vane, with the exception of Poncelet undershot wheels, which will be discussed separately. These wheels are of two kinds:

(1.) Wheels where the water acts partly by weight, partly by impulse.

(2.) Wheels where the water acts solely by impulse.

In the first class of wheels the proportion of the fall which acts by impulse to the whole fall is determined by the following considerations:

(1.) The greater the speed of the wheel the less will be the breadth of crown necessary to carry the quantity of water, and therefore the less the first cost of the wheel.

(2.) The greater the speed of the wheel the greater must be the ratio of that part of the fall which acts by impulse to the whole fall, and therefore the less the efficiency.

It is found that practically the velocity of the shrouding of overshot and high breast wheels cannot exceed about 6 feet per second without causing the water to splash over the edges of the buckets. Since the best velocity of the perimeter is about equal to half the velocity of impact, the greatest velocity of impact ought not to exceed 12 feet per second, which is due to a fall of about 2 feet 3 inches. In the case, therefore, of high falls no material increase of efficiency can be obtained by diminishing the velocity; where, however, the fall is low, a very great increase of efficiency can be obtained by diminishing the velocity of impact. The following table gives the theoretical and estimated actual efficiency of breast wheels for falls ranging from 3 to 12 feet, corresponding to velocities of impact due to 1 foot and 2 feet respectively, 6 inches being deducted for clearance at the tail. The theoretical values have been calculated on the assumption that the efficiency of that part of the fall which acts by impulse is equal to ·4, and that the effective power developed by the part which acts by weight only is equal to the gross power. The estimated actual values are based on the assumption that the same relation between actual and theoretical values holds good for each fall.

For falls of over 10 feet the experimental co-efficient of efficiency, with the ordinary velocity of periphery, is about ·7. The table below gives ·86 as the theoretical efficiency, and therefore the ratio actual to the theoretical will be equal to ·82. This co-efficient will give too high estimated values for the low falls, but sufficiently near the truth for the sake of illustration.

Height due to Velocity of Impulse.	3	4	5	6	7	8	12	16	Falls.
One foot .	·63	·72	·78	·81	·84	·86	·91	·93	Theoretical Efficiency.
	·52	·59	·64	·66	·69	·70	·75	·76	Estimated actual.
Two feet .	·43	·58	·66	·72	·76	·79	·86	·89	Theoretical Efficiency.
	·35	·48	·54	·59	·62	·65	·70	·73	Estimated actual.

In the case of low breast wheels the velocity of the perimeter is not limited by the same consideration, since the shrouding is surrounded by a breast which prevents the escape of the water. The determination of the velocity depends without limitation solely on the two conditions already stated. By the aid of these two conditions we may, according to circumstances, determine whether economy should be solely consulted at the expense of efficiency by adopting an impulse wheel, or regard be had only to efficiency by adopting a weight and impulse wheel with the least possible velocity of perimeter.

In the case of overshot wheels the water begins to drop from the buckets very soon after they dip below the axis of the wheel. Since it is impossible to ascertain the quantity which runs out of each bucket at each successive interval in the descent, it is impossible to calculate theoretically the efficiency of an overshot wheel. Their efficiency is manifestly much less than that of breast wheels.

The breadth of the crown of a high breast wheel must be determined as follows. If D be the diameter of wheel in feet, θ the angle subtended at the centre by the full buckets, h the height due to the velocity of impulse, and K the ratio of the speed of the wheel to the velocity of impulse, the time of describing the arc of full buckets will manifestly be equal to $\frac{D\,\theta}{2\,K\,\sqrt{2\,g\,h}}$, and if Q be the quantity of water discharged in cubic feet per second, the quantity in the buckets at any instant will be equal to $\frac{Q\,D\,\theta}{2\,K\,\sqrt{2\,g\,h}}$. If, therefore, b be the breadth of the crown, t the thickness of a rim of water spread equally over the crown so as to equal the quantity in the buckets, we must have

$$\frac{D\,\theta\,b\,t}{2} = \frac{Q\,D\,\theta}{2\,K\,\sqrt{2\,g\,h}},$$

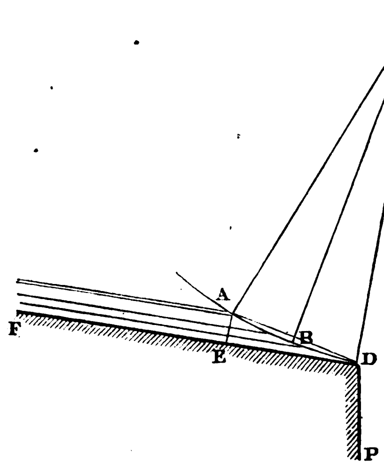
or

$$b = \frac{Q}{t\,K\,\sqrt{2\,g\,h}};$$

the value of t depends upon the shape of the buckets; it will rarely exceed half the depth of the crown.

The breadth of an impulse wheel is determined solely by the size of the aperture necessary to discharge the given quantity of water, and may therefore be materially diminished by a slight increase in the depth of that aperture. It has already been shown that the diameter of a breast wheel ought to be so chosen that the point in the circumference at which the water strikes the wheel is not at a less distance from the summit than the point at which the breast terminates is from the lowest point. That of undershot impulse wheels, with flat radial vanes, may be determined as follows:

Fig. 5.



1st. Undershot wheels with vanes working clear of tail water.

Let F E D represent the wheel race, which may be either horizontal or slightly inclined to the horizon.

This inclination is given in order that the velocity of the jet, when it leaves the sluice, may not be lessened by friction when it strikes the vanes. Since this ought, therefore, to be what is called the train inclination due to that velocity, its rate will increase with the increase of head, and it is plain that the velocity of the jet on issuing from the aperture will be exactly equal to the velocity with which it would have struck the vane if it had moved along a horizontal race of the same length. Therefore the only way in which loss of head from this cause can be avoided is to put the opening of the sluice close to the vanes.

In order to let as little of the water as possible escape without striking the vanes these ought to be as close together as possible, and the axis ought to be so situated that the outside periphery of the vanes is tangential to the wheel race. Also in order to avoid back pressure it is manifest that this point of contact ought to be the bottom of the race.

Through D, the bottom of the race, draw D O at right angles to F D, then the centre of the axis ought to be in this line. Take O for the centre. Draw the arc D B A meeting the surface of the jet in A, join O A and draw O B bisecting the arc A D in B. Now the theoretical effective power developed by any vane at any instant will be to the whole power of the water striking it in the ratio $\frac{\cos^2 \alpha}{2}$, where α is the angle between a normal to the vane and the direction of the jet, and is therefore equal to the angle at the centre subtended by the arc between the lowest point D and the bottom of the vane. Since the distance of B from the bottom of the race is only one-quarter of the depth of the current, fully three-quarters of the whole flow would strike the vane in this position if the interval between the vanes were not less than half the arc A D. If, therefore, the diameter of the wheel be so designed that only a certain percentage of the power which acts against the vane in this position shall be lost by obliquity, the total percentage lost from the same cause will not differ perceptibly from this amount. Draw A E at right angles to F D, then A E will be equal to the depth of the current. Let R, D, t represent respectively the radius of the wheel, the length of the arc sunk in the current, and the depth of the current in feet. Draw the chord A D. Then the angle A D E will be equal to the angle B O D. Its magnitude is always so small that its sine is equal to its circular measure nearly. Therefore

$$\begin{aligned} D &= R \times \text{circular meas. of } \angle AOD \\ &= R \times \text{twice the sine of } \angle ADE \\ &= \frac{2\,R \cdot AE}{AD}. \end{aligned}$$

Since the chord A D is practically equal to the arc A D, we have the following relation between the three quantities:

$$D^2 = 2\,R\,t.$$

Since this relation is independent of the velocity of the current, the height of the fall has nothing to do with the diameter of the wheel. So far as efficiency is concerned the effective power lost by the obliquity of the vane in the middle position to the gross power of the water striking it is in the ratio of $\frac{1 - \cos^2 \alpha}{2} : 1$, in which $\frac{1 - \cos^2 \alpha}{2} = \frac{\sin^2 \alpha}{2} = \frac{AE^2}{2\,AD^2} = \frac{t^2}{2\,D^2}$.

If, therefore, p be the maximum admissible rate per cent. of loss of head due to obliquity we shall have

$$\frac{t^2}{2\,D^2} = \frac{p}{100}.$$

Substituting for D its value we get

$$p\,R = 25\,t.$$

The following table gives the diameters and lengths of perimeters immersed corresponding to a few different depths of currents and different values of p.

Percentage lost by Obliquity.	Depth of Current.				
	" 4	" 6	" 9	" 12	—
2	' " 8·4	' " 12·6	' " 18·9	' " 25·0	Diameter.
	1·8	2·6	3·9	5·0	Periphery immersed.
4	4·2	6·3	9·4	12·6	Diameter.
	1·2	2·1	2·8	3·6	Periphery immersed.
8	2·1	3·2	4·8	6·3	Diameter.
	0·10	1·6½	1·11	2·6	Periphery immersed.

Since the diameter of wheels of equal efficiency are directly as the depth of the current, which may be varied at will, the diameter of a wheel of this class may be designed to suit the speed of various kinds of machinery by direct action, without affecting the efficiency, and by this means the heavy loss of power due to the friction of intermediate gearing may be avoided.

(2.) Undershot wheels with sunk vanes.

The same formulæ apply to this class of undershot wheels, t in this case representing the depth of the periphery below the surface of the current.

There is another class of undershot wheels with curved vanes, invented by Poncelet, which requires separate investigation. The vanes are curved in such a manner, that a tangent to the curve made by a vertical section at the lowest point is parallel to the relative velocity of the water and the vane, the tangent to the upper edge being radial.

Of these wheels Professor Rankine remarks, that the water finally glances off the vane in a direction relatively to the vane, parallel, but opposite to that in which it glanced on to the vane, so that the final absolute velocity will be radial, if we look upon the vane's motion in the interval as wholly linear at right angles to the radius through the point of impact. Therefore, in accordance with his rule that "the energy exerted by the water on the vane *in direction of motion* is equal to the energy lost by the water," the efficiency will be equal to $\cos^2 a$, where a is the angle between the direction of the jet and the vane, friction being left out of consideration. The words in italics are added by the Author. They are necessary to make the statement a faithful interpretation of the equation

$$P'u = \frac{W(v_1^2 - v_2^2)}{2g}.$$

This is the accepted theory of the Poncelet wheel. To illustrate it more clearly, let us consider the action of an indefinitely small particle of water of mass δm, and in order to simplify the action of gravity, let us suppose the motion of the vane horizontal. Let a velocity $\frac{v_1 \cos a}{2}$ be impressed on both the vane and the particle in a direction opposite to that of motion, so as to bring the vane to rest. If neither friction nor curvature cause loss of velocity, the particle will rise to the height due to the relative velocity, and at that instant its relative velocity will be nil and absolute velocity equal to $\frac{v_1 \cos a}{2}$. If, then, $c v_1^2$ represent the energy communicated to the vane, we shall have in accordance with the theory just enunciated, since the energy lost by the water has been exerted in imparting energy to the vane and overcoming the resistance of gravity,

$$c v_1^2 + \delta m g H = \frac{\delta m v_1^2 \left(1 - \frac{\cos^2 a}{4}\right)}{2};$$

where H is the height due to the relative velocity. The particle will then begin to descend, and its relative velocity, when it reaches the bottom of the vane, will be equal to the initial relative velocity in magnitude, but will be opposite to it in direction, so that the final absolute velocity will be simply $v_1 \sin a$ vertical, and if $c'v_1^2$ represent the energy imparted to the vane during the descent, we shall have

$$c' v_1^2 - \delta m g H = \frac{\delta m v_1^2 \left(\frac{\cos^2 a}{4} - \sin^2 a\right)}{2};$$

therefore $$(c + c') v_1^2 = \frac{\delta m v_1^2 \cos^2 a}{2}.$$

If for H we substitute its value

$$\frac{v_1^2 \left(\frac{\cos^2 a}{4} + \sin^2 a\right)}{2g}$$

we get $$c v_1^2 = c' v_1^2 = \frac{\delta m \cos^2 a}{4};$$

so that the energy imparted during the ascent has been equal to the energy imparted during the descent. The whole energy imparted will therefore, theoretically, be equal the sum of the energies exerted by the separate particles; but when we consider the action of the whole together, it is evident that only those particles which strike the vane first will reach the top, the subsequent particles being obstructed in their ascent by the return of the first, and the particles which first reach the top will not act on the vane on their return, being forced from it by the upward motion of the succeeding ones. So that the energy imparted to the vane due to both direct action and reaction will be much less than that assigned by theory in the case of one particle.

In order to eliminate the effect of the action of gravity, we will suppose that the axis of the wheel is vertical, so that the motion of the jet is horizontal. At the point where the tangent to the vane is radial, since the relative velocity is unaffected by the impulse, the absolute velocity will be equal to

$$v_1 \sqrt{\frac{\cos^2 a}{2} + \sin^2 a},$$

and therefore the energy lost by the water and imparted to the vane will be equal to $\frac{\delta m v_1^2 \cos^2 a}{4}$. Imagine now another Poncelet vane added to this with its radial end meeting this tangentially. The relative velocity when it reaches the other end of the vane will be unaltered in magnitude, but opposite in direction, so that the final absolute velocity will be equal to $v_1 \sin a$, and therefore the energy lost by the particle of water during contact with the second half of the vane will also be equal to $\frac{\delta m v_1^2 \cos^2 a}{4}$. Since the flow is continuous and the water discharged freely from the opposite end of the vane, one particle of water does not interfere with another, and the efficiency of such a compound vane is theoretically equal to $\cos^2 a$.

We are now in a position to discuss the two turbine theories. According to one the turbine is simply an impulse wheel, according to the other a pressure and impulse wheel.

If a turbine be viewed as an impulse wheel only, it is immaterial what the shape of the vanes may be, so long as there are no re-entering angles, provided that they are tangential to the relative velocity on the receiving side and their final relative direction is such that the final velocity is a minimum, if the theory expressed by the equation

$$P u = \frac{W (v_1^2 - v_2^2)}{2 g}$$

be true. But if, on the contrary, the effective power imparted is not equal, but only proportional to this, it follows that the shape of the vane is of the utmost consequence. This last is the theory maintained by the Author and fully discussed in Part II.

According to the other theory the power is imparted to the turbine partly by the statical pressure of the fluid, partly by change of vis viva. Thus if p be the pressure of the fluid on leaving the guide blades, ω, the weight of an unit of volume of the water, using the symbols explained in Part II., we have for the gross power of the fall

$$\omega Q H = \left(\frac{p}{\omega} + \frac{v_1^2}{2g}\right) \omega Q;$$

and if p', v_0 represent the pressure and velocity of the water on leaving the turbine, the power lost will be equal to

$$\omega Q H - \left(\frac{p'}{\omega} + \frac{v_0^2}{2g}\right) \omega Q = \left(\frac{v_1^2 - v_0^2}{2g} + \frac{p - p'}{\omega}\right) \omega Q.$$

In order to ascertain the value of the final absolute velocity v_0, the turbine is supposed to be reduced to rest by applying both to it and the water a velocity at every point equal and opposite to that of the vane. The relation between the initial and final relative velocities and pressures is given by the theorem of Bernouilli, which may be thus enunciated (Viry, "Cours de Mécanique," Tome IV.):—

If a liquid be moving with a steady motion along a pipe of a variable section, but varying in such a manner that the changes of section are gradual (so that there is no agitation in the water), then, if in addition the friction of the molecules between each other and against the sides of the pipe be neglected, Bernouilli's theorem states that—

The difference between the heights due to the velocities at each section is equal to the difference of level of the centre of gravity of the two sections, increased by the difference between the heights due to the pressures.

If, then, u_1, u_0 represent the initial and final relative velocities we shall have in the case of a parallel-flow turbine, where h is the depth of the revolving drum,

$$\frac{u_0^2 - u_1^2}{2g} = \frac{p - p'}{\omega} + h.$$

In the case of an outward and inward flow,

$$\frac{u_0^2 - u_1^2}{2g} = \frac{p - p'}{\omega}.$$

Now the value of u_1 is known directly in terms of the given values of v_1 and v the velocity of whirl of the receiving side of the drum, and the angle α between their directions, therefore the above equations give the value of u_0 in terms of the same known quantities, and by this means we can determine the value of the final absolute velocity v_0, the angle β between the direction of whirl and the final tangent to the vane being quite arbitrary, except that it must satisfy this relation—

$$b\, v_1 \sin \alpha = b'\, u_0 \sin \beta$$

in the case of parallel-flow turbines, where b, b' are the initial and final breadths of the crown, and in the case of outward-flow turbines, the relation

$$R_1\, D_1\, v_1 \sin \alpha = R_0\, D_0\, u_0 \sin \beta.$$

This condition is simply the analytical expression of the statement, that the quantity of water which enters the turbine is equal to the quantity which leaves it, the spaces between the vanes according to this theory being always full.

The Author will endeavour to show—

1st. That no such statical pressure could exist in a turbine;

2nd. That no turbines are in reality designed on the supposition that such a pressure exists.

As to the first: Since the pressure must be the same at every point, whether the turbine be in motion or reduced to rest by impressing equal and opposite velocities on the water and turbine, it follows that the pressure is unaffected by the motion, and that therefore it either remains constant to the end, or is converted into vis viva, which is itself likewise unaffected by the motion through the turbine, and therefore the whole head due to the pressure is either lost or is utilised on issuing from the vanes in the same way as in the case of a reaction wheel. In a turbine the whole of the useful effect is obtained before the water issues from the vanes. As to the second: By means of Bernouilli's theorem, Viry has shown that if p_a, p_t represent the pressure due to the atmosphere and tail water,

$$v_1^2 = 2g\left\{H + \frac{p_t + p_a}{\omega} - \frac{p}{\omega}\right\}$$

also in the case of parallel-flow turbines

$$\frac{p}{\omega} = \frac{p_t + p_a}{\omega} + H\left\{1 - \frac{b' \sin\beta}{b \sin 2\alpha}\right\}$$

and in the case of the Fourneyron

$$\frac{p}{\omega} = \frac{p_a + p_t}{\omega} + H\left\{1 - \frac{D_0 R_0^2 \sin\beta}{D_1 R_1^2 \sin 2\alpha}\right\}$$

the factor $R_0^2 \div R_1^2$ being due to the supposed existence of a centrifugal force.

Viry divides his investigation into two heads:—

1st. Given a theoretically perfect turbine to find the discharge and the efficiency.

2nd. Given the discharge and the height of the fall to design a theoretically perfect turbine capable of passing the water.

Towards the end of his investigations into the final phase of the problem, he states that the internal pressure p must be greater than the external pressure $p_a + p_t$ to prevent the tail water rushing into the turbine through the space between the fixed and revolving drums. It is strange that the thought never struck Viry, that where the tail water could rush in, the water within the turbine could rush out, if the internal pressure were greater than the external.

In completing the second part of his investigation, it is necessary to determine the values of b and b', D_1 and D_0. This he does by putting $1 - \frac{b' \sin\beta}{b \sin 2\alpha}$ and $1 - \frac{D_0 R_1^2 \sin\beta}{D_1 R_0^2 \sin 2\alpha}$ respectively equal to zero. This simply means that the pressure within the turbine must be equal to the pressure without, and that the initial velocity v_1 must be equal to $\sqrt{2gH}$, that is, it must be the same as if discharged freely into the open air. Turbines, then, are simply impulse wheels.

There is yet another point to be investigated, viz., to what extent is the co-efficient affected by a departure from the best velocity of the wheel. Although the theoretical and experimental values of the best ratios do not coincide, the magnitude of the range of values corresponding to a given decrease of efficiency determined from theory will probably not differ much from that ascertained by experiment.

The formula for undershot wheels with sunk vanes is

$$\text{Efficiency} = \frac{(v^2 - u^2)\,u}{v^3} = (1 - K^2)\,K,$$

where $K = \frac{u}{v}$, now the theoretical best efficiency is ·385, corresponding to the value ·58 of K; but the value of the expression $(1 - K^2) K$ is never less than ·357, or the variation from best efficiency does not equal 3 per cent., for values of the ratio varying from ·4 to ·7.

The formula for undershot wheels with vanes working clear of tail water is

$$\text{Efficiency} = \frac{2(v-u)u}{v^2} = 2(1-K)K,$$

the value of the best efficiency being ·5 corresponding to the value ·5 of K. The variation in this case from best efficiency will not exceed 3 per cent. for values of K from ·38 to ·62.

The Author has not been able to find any statement of the result of experiments to ascertain the actual efficiency of wheels with flat vanes working clear of the tail water like those of Smeaton for drowned vanes. If we use the same value of the ratio efficiency of wheel ÷ theoretical efficiency of vane, viz., ·77, we shall get ·38 as the experimental efficiency of these wheels. It has been shown that ·8 is a fair experimental value of the factor by which the theoretical value of the efficiency of breast wheels ought to multiplied, and we are now, therefore, in a position to decide upon the best class of wheel to use in any case.

When the choice lies between an undershot wheel with vanes drowned and vanes working clear, we may determine the limit of equal efficiency thus: Let H be the height of the fall at which equal efficiency is obtained, and h the clearance at the tail in the case of the wheel working with its vanes clear of the tail water, then

$$\cdot 3\,H = \cdot 38\,(H - h);$$

if we put $h = 4$ inches, we get $H = 1'\,7''$.

Now 4″ is the least value we can give for the clearance at the bottom of the race, and therefore for falls of less than 18″ an undershot wheel with drowned vanes should be used.

In the case of weight and impulse and impulse wheel, if the point of impact in the first case be only 1 foot below head water the equation for determining the limiting position is

$$\cdot 38\,(H-h) = \cdot 8\,\{(H-h-1) + \cdot 4\}$$

or with the same value of h,

$$H = 1\cdot 47.$$

If the point of impact be 2 feet below head water, the equation will be

$$\cdot 38\,(H-h) = \cdot 8\,\{(H-h-2) + \cdot 8\}$$

or

$$H = 2\cdot 7.$$

So far then as efficiency alone is concerned, it would seem that the choice lies only between breast wheels and wheels with submerged vanes; but there are two other elements in the question, viz., first cost, and loss of efficiency by the flooding of the tail race. Now in the case of very low falls the difference between the efficiencies of breast wheels and undershot wheels is very slight, whilst the difference in the first cost is very great, and it is precisely in these cases, which usually occur at weirs in large rivers, that the tail race is most frequently and deeply drowned by floods. Although loss of efficiency by resistance from the tail water may be about the same in the two cases, the quantity of flood water acting against the undershot wheel may be increased to any amount so as to make up for loss of efficiency, whilst only a limited quantity can be let on to the breast wheel. Owing to these two causes, impulse wheels ought to be used for less falls than 3 feet 6 inches.

To fill this gap, then, it is plain that we ought to use a wheel whose efficiency is unimpaired by the rising of the tail water, and the only wheels capable of fulfilling this condition are turbines. Since these wheels are much cheaper than breast wheels, if they were of equal efficiency it would follow that they ought in all cases to be used for falls over 2 feet. The conclusion, however, at which the Author of this Paper has arrived is that the efficiency of the best constructed turbines cannot much exceed ·50, and therefore we ought to adopt:

(1.) For falls under 2 feet, an undershot wheel with sunk vanes.

(2.) For falls from 2 to 5 feet, a turbine.

(3.) For falls from 5 to 12 feet, a breast wheel for the summer flow, supplemented by a turbine in flood time.

(4.) Above 12 feet, a breast wheel only, unless the fall be so great that the first cost would put a breast wheel out of question. In that case a turbine ought to be adopted.

PART II.

Principles of Construction and Efficiency of Turbines.

In the following investigation the head of water referred to is the nett head at the point, where the water is discharged from the guide blades in the supply chamber against the vanes of the revolving drum after all losses of head due to its passage through the supply pipes and the supply chamber have been deducted ; in other words, the height due to the velocity with which the water would issue freely into the air from the guide blades.

It has been shown in Part I. that when a jet strikes a curved vane, moving in any assigned direction, the vane ought to be of such a shape that the absolute path is circular; that the tangent to the vane at the point where the jet strikes it should be parallel to the relative motion of the jet and vane, and the tangent to the vane at the point where the jet leaves the vane should be parallel to the direction of motion of the vane if friction be left out of consideration.

Since the quantity of water passing every point of the vane at the same instant is constant, the area of the jet estimated at right angles to the direction of absolute velocity must be inversely as the absolute velocity, and estimated at right angles to the relative path must be inversely as the relative velocity. It follows, therefore, that when two or more vanes similar and equal in every respect, and similarly situated with respect to the directions of motion of the jet and vane, are moving in any assigned direction, the area of the space between the two can never be less than $\frac{a\,v_1}{v}$, where a is the area of the section of the jet at the instant it leaves the guide blades, v_1 its initial absolute velocity, and v the relative velocity at any instant, if the required condition, viz., that the initial velocity on leaving the guide blades should be the same as if the jet discharged freely into the open air, is complied with.

If the area at any point is less than this the initial velocity will be less, since the total quantity of water discharged will be less, whilst the area of discharge at the guide blades remains constant. Consequently the nett head at the orifice of the guide blades will be expended partly in producing velocity, partly in producing pressure against the vanes between the point where the jet strikes them and the point to which v refers. Now the head due to the pressure against the vanes is wholly lost, since the pressure is exerted equally against both vanes, and therefore tends as much to retard as to accelerate the motion of the vanes. The same remarks apply to the absolute path of the water.

In the investigation of the case of a single vane the jet was supposed to move parallel to itself in the direction of motion so as to strike the vane at the same point at each instant. In turbines there are a succession of jets immediately following each other, the direction of the absolute velocity of each jet being inclined at a constant angle to the direction of absolute velocity of the vanes. It is evident that this corresponds exactly with the condition laid down in the case of a single jet and a single vane, provided there are also a succession of vanes equal in number to the jets, and the problem we have to solve is therefore what must be the shape of the vane in order that:

(1.) The tangent to the vane at the point of impact shall be parallel to the relative motion of the jet and vane.

(2.) The absolute path shall be circular.

(3.) The absolute velocity shall never be less than $\frac{a\,v_1}{a'}$, where a is the area of the jet, v_1 the velocity of discharge into the air, and a' the area of the absolute path at right angles to the direction of motion of the jet.

Since the vanes are similarly situated with respect to the direction of motion, if that motion be linear the interval between the vanes estimated parallel to the direction of motion will be constant, and therefore the area at right angles to the absolute velocity will increase until the direction of absolute velocity is at right angles to the direction of motion, and will then begin to decrease. Therefore the minimum possible magnitude of this final absolute velocity must occur when the direction of that final absolute velocity is at right angles to the direction of motion. Since the areas at right angles to the direction of motion are all equal, this minimum final velocity can never be less than the component of the initial velocity at right angles to the direction of motion; also, if this final velocity be equal to the above component of the initial velocity, this component will have the same value at every intermediate point, since the vane cannot have exerted any force on the jet to give this component either a maximum or a minimum value between the instant of impact and of finally leaving the vane. Therefore, in order to obtain a maximum of efficiency, since this corresponds, *cœteris paribus*, with a minimum final velocity, we must so design the vanes that the component of the absolute velocity at right angles to the direction of motion is equal to the same component of the initial velocity.

In the illustration given above of the problem we have to solve the path of the vanes has for simplicity's sake been supposed rectilinear. In the case of turbines it is in reality circular. The effect of this curvilinear motion will be discussed in each case separately. They will be taken in the following order:

(1.) Outward-flow turbines.
(2.) Inward-flow turbines.
(3.) Parallel-flow turbines.

The following symbols will be used to express the same elements in each case:

Q = quantity of water discharged per second in cubic feet;
H = whole head in feet;
v_1 = velocity of discharge from guide blades in feet per second;
a = angle between direction of jet and direction of motion;
δ = angle between radii through the point of impact and the point where the absolute path cuts discharging side;
R_1 = radius of receiving side of revolving drum;
R_0 = do. discharging do.;
μ = value of ratio $R_0 \div R_1$;
ρ = any intermediate radius between R_1 and R_0;
A = an area of section of jet parallel to direction of motion;
ω = angular velocity of drum;
D_1 = depth of drum parallel to the axis in cases (1) and (2); at receiving side;
D_0 = do. do. at discharging side;
d = any intermediate depth between D_1 and D_0;
r = radius of absolute path;

also let D_1, D_0, d represent depths of drum in parallel-flow turbines parallel to the axis at the circumference nearest to and most remote from the axis and at any intermediate point.

Fig. 7 (p. 42) represents part of a section at right angles to the axis through guide-blade chamber and revolving drum of an outward-flow turbine. O is the centre of the axis of the shaft. O B and O Q are the internal and external radii of the revolving drum. In these turbines the water flows outwards through the guide-blade chambers and strikes the vanes of the revolving drum in the direction A B indicated by the arrow, making the angle A B C with the direction of motion of the vanes, B C being a tangent to the common circumference of guide-blade chamber and revolving drum, and A B a tangent to the guide blade at a point where a guide blade meets that circumference. The angle a is therefore equal to the angle A B C, and the component of the initial velocity parallel to the direction of motion is $v_1 \cos a$, and at right angles to it $v_1 \sin a$. Produce A B to T. Join O T, and in O T produced take the point Q, such that T Q = B T. Through

B and Q draw Q O′, B O′, at right angles respectively to B T and T Q, intersecting each other in the point O′: with centre O′ and radius O′ Q = O′ B describe the arc B p Q. If the vane be so designed that B Q represents the absolute path, the conditions laid down will manifestly have been complied with, since the water initially moves tangentially to the absolute path, and finally radially at right angles to the direction of motion.

Since Q is the point of discharge, we have O B = R_1 and O Q = R_0. With centre O and radii, whose values lie between R_1 and R_0, describe a succession of arcs, such as p P, p' P′, &c., equidistant from each other, $p p'$ being points in the absolute path. Along these arcs measure off p P, p' P′, such that if ρ, ρ' represent the radii $o\,p$, O p' respectively,

$$p\,P = \frac{\omega\,\rho\,(\rho - R_1)}{v_1 \sin a};$$

$$p'\,P' = \frac{\omega\,\rho'\,(\rho' - R_1)}{v_1 \sin a};$$

then will the locus of all such points P, P′, &c., represent the shape of a vane which will cause the jet to describe the absolute path B p Q if the radial velocity remain constant and equal to $v_1 \sin a$, since a particle of water will reach the points p, p' in the absolute path simultaneously with the corresponding points P, P′, &c., of the relative path, $\omega\,\rho$, $\omega\,\rho'$, &c., being the velocities of points P, P′ parallel to the direction of motion and $\frac{\rho - R_1}{v_1 \sin a}$ $\frac{\rho' - R_1}{v_1 \sin a}$, &c., the time it takes the particle to move radially through the spaces $\rho - R_1$, $\rho' - R_1$, &c., in accordance with the assumption that the radial velocity is constant and equal to $v_1 \sin a$. If we take a point in the relative path at a distance from the centre equal to $R_1 + \delta\,\rho$, the chord joining the points, whose distances from the centre are R_1 and $R_1 + \delta\,\rho$ respectively, will be inclined to the direction of motion at an angle whose tangent is equal to

$$\frac{v_1 \sin a}{v_1 \cos a - \omega\,(R_1 + \delta\,\rho)},$$

when $\delta\,\rho$ is indefinitely diminished. This chord becomes ultimately initial tangent to the relative path, and the tangent of its inclination to the direction of motion will be equal to

$$\frac{v_1 \sin a}{v_1 \cos a - \omega\,R_1},$$

or the vane initially will be parallel to the relative direction of the motion of the jet and vane. If we impress upon each particle of the water and of the vane at any instant the absolute velocity of the vane in an opposite direction so as to bring the vane to rest, the components of the final velocity of the water will be radially $v_1 \sin \alpha$, and tangentially ωR_0; and therefore the tangent to the relative path at the point where the jet leaves the vane will be inclined to the direction of motion at any angle whose tangent is equal to

$$\frac{v_1 \sin \alpha}{\omega R_0},$$

which is in all cases very small; so that a vane constructed in the manner shown very nearly complies with the second condition of maximum efficiency for correct vanes, viz., that the tangent to the relative path at the point where the jet leaves it should be parallel to the direction of motion of the vane.

Since the motion is curvilinear, sections through the jet at successive points of the vane parallel to the direction of motion will not be parallel to the first similar section at the point of contact of vanes and guide blades, but will be inclined to it at an angle θ, which is equal to the angle between the radii passing through the points where the initial section and section referred to cut the vane. If ϕ be the angle between the radius vector, and the tangent to the absolute path at any point, the angle between the tangent to the absolute path and the initial radius will manifestly be equal to $\theta + \phi$, since the radius vector always lies between initial radius and the tangent. Therefore if $v_1 \sin \alpha$ remain constant, the absolute velocity at any point (ρ, θ) in the absolute path, taking centre O for origin, and initial radius as the initial line, will be equal to $v_1 \sin \alpha \sec (\theta + \phi)$; and therefore the radial velocity at the same point will be equal to $v_1 \sin \alpha \sec (\theta + \phi) \cos \phi$, which is greater than $v_1 \sin \alpha \sec \theta$, since $\cos \phi \cos \theta$ is greater than $\cos (\theta + \phi)$. Therefore the radial velocity will always be greater than $v_1 \sin \alpha \sec \theta$, if loss of velocity due to friction and curvature of absolute path be left out of consideration, until we reach the point of discharge, at which the direction of the absolute velocity is at right angles to the direction of motion, and the final absolute velocity therefore equal to $v_1 \sin \alpha \sec \delta$, so that the final area need not exceed A $\cos \delta$. Since, however, both friction and curvature cause loss of velocity, it is evident that the final area must be greater than this, but how much greater it is impossible to ascertain. The condition that the initial velocity should be the same as if the jet were discharged into the open air will be complied with if the area of each section be not less than $\frac{A\, v_1 \sin \alpha}{v}$, where v is either the absolute or relative velocity according as the section referred to is at right angles to the absolute or relative velocity. It does not matter how much it may exceed that amount.

In outward-flow turbines, if the depth parallel to the axis be constant, the area at the point of discharge will be sufficient if the loss of radial velocity does not exceed $v_1 \sin \alpha \left(1 - \frac{R_1}{R_0}\right)$. A common value of the angle α is 20°, and the value of the ratio $\frac{R_0}{R_1}$ is never less than 1·3. This loss therefore would correspond with a loss of about 8 per cent. of the whole initial velocity, an amount much in excess of what would really be incurred. Since the vanes recommended in this Paper are designed on the assumption that, owing to the resistance of friction and curvature, the radial velocity remains constant and equal to the initial radial velocity $v_1 \sin \alpha$, the final loss of velocity would be equal to $v_1 \sin \alpha (\sec \delta - 1)$, which for ordinary values of α and δ amounts to about 2 per cent. of the whole initial velocity.

We have next to investigate what will be the probable effect of a variation of the radial velocity from the observed constant value $v_1 \sin \alpha$. At the instant the jet strikes the vane the radial velocity will be equal to this, whatever its subsequent changes may be; therefore the tangent to the vane initially will always be parallel to the relative velocity of the water and the vane, so long as the ratio $v_1 \div \omega R_1$ remains constant. If the average radial velocity exceeds $v_1 \sin \alpha$, the effect will be that the successive particles of water will reach any point P in the relative path before the point P reaches the corresponding point p in the absolute path, so that the curve of the absolute path actually traversed will be wholly within and on the concave side of the absolute path B p Q, and will cut the external circumference at an acute angle, the final absolute velocity being divisible into two components; one at right angles to the direction of motion, and the other parallel to it in the direction opposite to that of motion. The final absolute velocity cannot, however, be much greater than $v_1 \sin \alpha$, since the radial velocity must be less than $v_1 \sin \alpha \sec \delta$, so that the effective power of the vanes will not be appreciably affected if the average radial velocity be somewhat greater than $v_1 \sin \alpha$.

If, on the contrary, the average radial velocity be less, the successive particles of water will not reach any point P in the relative path till after the point P has reached the corresponding point p in the absolute path, so that the curve of the absolute path actually traversed will lie wholly without on the convex side of the absolute path B p Q, and will cut the external circumference at an acute angle, the final absolute velocity being divisible into two components; one at right angles to the direction of motion, and the other parallel to it in the same direction.

If the final absolute velocity does not differ much from $v_1 \sin \alpha$, the effective power developed will not be appreciably affected, but it will be more injuriously affected by a decrease in the average radial velocity than by an increase, because the final absolute velocity in the former case may be greater than in the latter, although the loss of velocity due to friction has been greater for the following reason:—If the average radial velocity be less, the angle between the tangent to the absolute path and the corresponding radius vector will never pass through the value zero, and if v_0, ϕ_0 be the values of the velocity parallel to the initial velocity $v_1 \sin \alpha$ and this angle at the point of discharge, the final absolute velocity will be equal to $v_0 \sec (\theta_0 + \phi_0)$. If, on the contrary, the average radial velocity be greater than $v_1 \sin \alpha$, the angle ϕ will pass through the value zero before the point of discharge, and the final absolute velocity will be equal to $v_0 \sec (\theta_0 - \phi_0)$, where, in addition, θ_0 is less than in the first case, so that, although v_0 in the latter is greater than v_0 in the former, $v_0 \sec (\theta_0 - \phi_0)$ in the latter case may be less than $v_0 \sec (\theta_0 + \phi_0)$ in the former, and the ratio effective power $\div$ power lost must be greater in the latter case, since there has been less loss due to friction. For this last reason, the efficiency may be greater than when the radial velocity is constant and equal to $v_1 \sin \alpha$: such excess, however, can only equal some fraction of $\frac{1}{2}$ per cent.

We have now on these data to determine the relations which subsist between the different elements of the turbine. The angle α is perfectly abitrary. The less it is the greater will be the efficiency of the turbine, but the efficiency which, on comparing the two relative efficiencies, may be taken to be proportional to $\cos^2\alpha$, is increased very little by a considerable change in the value of α, whilst the discharging power varies directly as the circular measure of α, since $\sin \alpha = \alpha$ nearly. Thus the discharging power of a turbine in which the angle α is equal to 10° is only about half that of a turbine of the same size in which the angle α is equal to 20°, whilst the efficiency of the latter would be to that of the former as $\cos^2 20 : \cos^2 10° = 89 : 97$. The area of discharge from the guide-blade chamber at right angles to the guide blades is equal to n A sin α, where n is the number of openings between the guide blades. If therefore c be the co-efficient of contraction, we must have

$$c\, v_1\, n\, A \sin \alpha = Q.$$

Before we can ascertain the value of A in terms of R_1 and D_1 it will be necessary to investigate the mode of constructing the guide blades, since A is equal to the area of the jet, as it issues from the guide blades, multiplied by cosec α.

Through O draw O D at right angles to A B, meeting A B in D. Then will O D be equal to $R_1 \cos \alpha$, and D B to $R_1 \sin \alpha$. With centre O and radii equal to $R_1 \cos \alpha$ and $R_1 \sin \alpha$ respectively draw two circles. The tangent to any guide blade at the point where it cuts the outer circumference of the guide blade chamber will touch the circumference of the circle radius $R_1 \cos \alpha$, and the normal to the guide blade at the same point will touch the circumference of the circle radius $R_1 \sin \alpha$.

Divide the circumference of the circle radius $R_1 \sin \alpha$ into n equal divisions at the points b, b', b'', &c.; with each of these points as centre and radius $R_1 \cos \alpha$ describe a succession of circular arcs cutting the outer circumference of the guide-blade chamber in a succession of points, B, B′, B″, &c. The angle between the tangents to the arc and the circumference at any of the points B, B′, B″, &c., will be equal to the angle α, therefore these arcs will be of a suitable shape for the guide blades close to the point of discharge. Now the maximum distance between two such consecutive circular arcs occurs when their radii coincide, and is equal to $\frac{2\pi R_1 \sin \alpha}{n}$, and this distance gradually decreases as their radii diverge from coincidence, therefore the maximum length of the arc suitable for a guide blade corresponds with the position of the describing radius where it passes through the points where the preceding arc cuts the outer circumference of the guide-blade chamber, since the area between two consecutive guide blades must be a minimum at the point of discharge. It is evident, however, that the guide blades must not terminate at this point, since they must overlap each other in order to give the proper direction to the jet. Beyond this point, towards the receiving side of the guide-blade chamber, the guide blades may be of any form, so long as they comply with the conditions that the sectional area between them at right angles to the direction of flow continually increases from the discharging towards the receiving side, and that the change of motion is gradual. If n be not less than $\frac{360}{\alpha}$, a suitable radius for the receiving side of the guide-blade chamber will be $R_1\,(2\cos\alpha - 1)$, so that the cylinder to which the initial tangents to the guide blades are tangential will bisect the width of the guide-blade chamber. With centre O, and any radius between $R_1\,(2\cos\alpha - 1)$ and $R_1 \sin\alpha$, describe a circle. With successive centres o' o'', &c., where the radii b' B', b'' B'', &c., cut the circumference of this circle, and radius equal to $R_1 \cos\alpha - b'\,o'$ complete the arcs of the guide blades until they meet the circumference, radius $R_1\,(2\cos\alpha - 1)$. The arcs so drawn will form suitable guide blades. Since the angle between any two consecutive radii, which describe the arcs on the discharging side, when they both pass through the same point, where a guide blade meets the discharging side of the guide-blade chamber, is so small that we may look upon the two as approximately coinciding, the minimum distance between the centres of two consecutive guide blades will be equal to $\frac{2\,\pi\,R_1 \sin\alpha}{n}$ approximately, and A will be equal to $\frac{2\,\pi\,R_1\,D_1}{n}$, less the thickness of a guide blade multiplied by cosec α.

In outward-flow turbines the direction of the flow of the water in the supply chamber is parallel to the axis of revolution. Hence, if R′ be the radius of the axis, or of any casing surrounding it, and $R = R_1\,(2\cos\alpha - 1)$ represent the inner radius of the guide-blade chamber, we must have

$$\pi\,(R^2 - R'^2)\,v = Q,$$

where v is the velocity of approach parallel to the axis, and therefore at right angles to the direction of the jet. We may therefore look upon the head due to this velocity as almost wholly lost, and the radius R must be so chosen as to reduce this loss to a minimum. Let the loss of head due to this cause be equal to p per cent. of the whole head. Then, on the supposition that the head due to the velocity of approach parallel to the axis is wholly lost, we shall have

$$v^2 = \frac{2\,p\,g\,H}{100}.$$

If p' represent the percentage of other losses of head due to friction in the supply pipes, &c., the nett head at the guide blades will be equal to

$$H\left(1 - \frac{p + p'}{100}\right),$$

and the discharge will be equal to

$$c\,(2\,\pi\,R_1\,D_1 \sin a - n\,t\,D_1)\,\sqrt{2\,g\,h\left(1 - \frac{p+p'}{100}\right)};$$

we must therefore have

$$\pi\,(R^2 - R'^2)\,\sqrt{\frac{2\,p\,g\,H}{100}} = c\,(2\,\pi\,R_1\,D_1 \sin a - n\,t\,D_1) \times$$

$$\sqrt{2\,g\,H\left(1 - \frac{p+p'}{100}\right)}.$$

Now the sum of the areas $n\,t\,D_1$ is very little removed from the area $\pi\,R'^2$, and we shall obtain a value for R on the safe side, if we neglect the loss of head $\frac{p+p'}{100}$ on the right-hand side of the equation. We shall then get

$$R_1 = \frac{20\,c\,D_1 \sin a}{\sqrt{p}\,(2 \cos a - 1)^2}.$$

For the sake of numerical illustration, we may take $a = 20°$ and $c = \cdot 8$, and we get

$D_1 = \cdot 2 R_1$	when $p = 2$
$D_1 = \cdot 28\, R_1$	when $p = 4$
$D_1 = \cdot 42\, R_1$	when $p = 9$
$D_1 = \cdot 57\, R_1$	when $p = 16$

The shrouding which covers the axis is frequently curved so as to direct the water towards the guide blades, thereby utilising a little of the velocity of approach in the guide-blade chamber. In order that the velocity parallel to the axis may remain constant, it is evident that the annular areas between the inner side of the guide-blade chamber and outer face of the shrouding must vary as the distance of each horizontal section from the bottom of the guide-blade chamber, since the quantity of water varies as that distance. Therefore if ρ be any intermediate radius of the shrouding, and d any intermediate depth, we must have

$$\frac{R_1^2\,(2 \cos a - 1)^2 - \rho^2}{R_1^2\,(2 \cos a - 1)^2 - R'^2} = \frac{d}{D_1}.$$

Values of ρ determined from this would be maximum values, and therefore give the limiting position of the curve of intersection of a vertical plane through the axis with the surface of the shrouding. The curve so determined would not, however, be a suitable one, because its convexity is towards the guide-blade chamber, and it does not meet the horizontal casing at the bottom of the guide-blade chamber tangentially. A suitable curve of intersection between the shrouding and a vertical plane through the axis is a parabola, whose vertex coincides with the intersection of the inner face of the guide-blade chamber and the shrouding, whose equation is

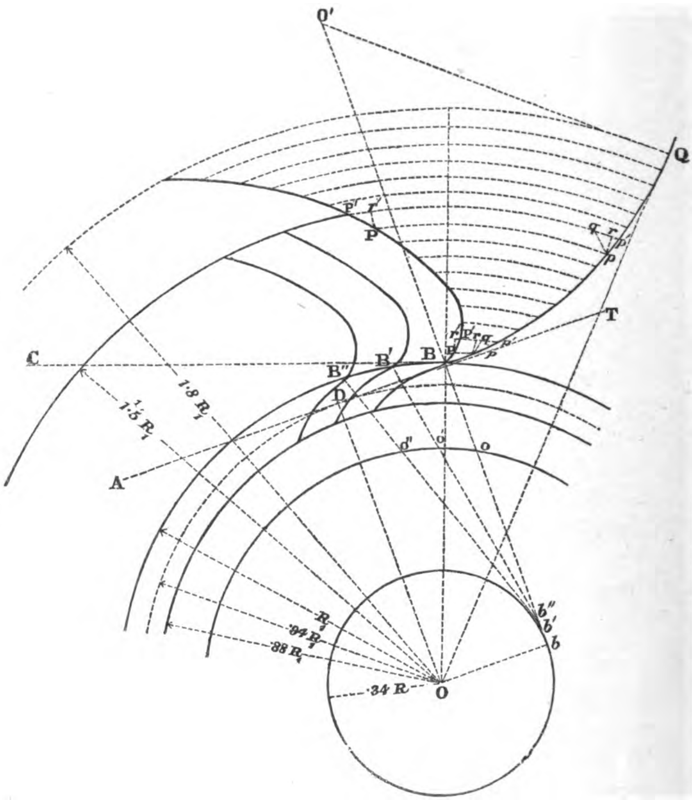
$$y^2 = \{R_1 (2 \cos a - 1) - R'\}^2 \frac{d}{D_1},$$

where

$$y = R_1 (2 \cos a - 1) - \rho.$$

We have now to determine the elements of the absolute path, in in terms of R_1 and R_0 and the angle a.

FIG. 7.



Since (Fig. 7) T is the intersection of the tangents to the curve of the absolute path at the points B and Q, where it cuts the receiving and discharging side of the drum, B T = T Q, also

$$B\,T = B\,O\,\frac{\sin B\,O\,T}{\sin B\,T\,O}$$

$$= \frac{R_1 \sin \delta}{\cos(\alpha + \delta)}$$

$$Q\,T = Q\,O - O\,T = Q\,O - \frac{B\,O \sin O\,B\,T}{\sin B\,T\,O}$$

$$= R_0 - \frac{R_1 \cos \alpha}{\cos(\alpha + \delta)}$$

therefore

$$R_1 (\cos \alpha + \sin \delta) = R_0 \cos(\alpha + \delta).$$

If we put

$$\delta = \frac{\pi}{2} - \phi$$

we get

$$\tan\frac{\phi}{2} = \frac{R_0 \tan\frac{\alpha}{2} + R_1}{R_0 + R_1 \tan\frac{\alpha}{2}}$$

$$\tan\frac{\delta}{2} = \frac{1 - \tan\frac{\phi}{2}}{1 + \tan\frac{\phi}{2}}$$

$$= \frac{\left(R_0 - R_1\right)\left(1 - \tan\frac{\alpha}{2}\right)}{\left(R_0 + R_1\right)\left(1 + \tan\frac{\alpha}{2}\right)}$$

also

$$O'\,B = B\,T \cot\frac{B\,T\,O}{2} = \frac{O\,B \sin B\,O\,T \cot\frac{B\,T\,O}{2}}{\sin B\,T\,O}$$

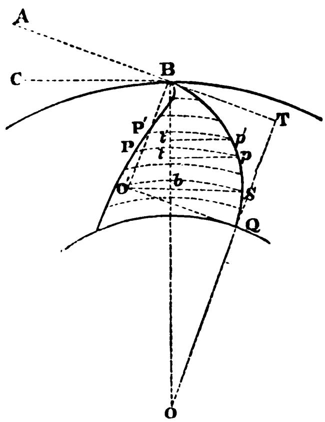
$$= \frac{O\,B \sin B\,O\,T}{2 \sin^2 \frac{B\,T\,O}{2}}$$

therefore

$$r = \frac{R_1 \sin \delta}{2 \sin^2\left(\frac{\pi}{4} - \frac{\alpha + \delta}{2}\right)} = \frac{R_1 \sin \delta}{1 - \sin(\alpha + \delta)}$$

The best values of the ratios $R_0 \div R_1$ and $\omega R_1 \div v$ will be investigated after the rules for designing inward and parallel flow turbines have been given.

FIG. 8.



Figs. 8 and 9 represent part of a section of an inward-flow turbine. O is the centre of the axis of the shaft. In these turbines the water flows inwards from a supply chamber surrounding the outer periphery of the guide-blade chamber, and strikes the vanes of the revolving drum in the direction A B, indicated by the arrow, making the angle A B C with the direction of motion of the vanes. B C being a tangent to the common circumference of the guide-blade chamber, and the revolving drum and A B a tangent to a guide blade at a point where it meets that circumference, so that A B C is equal to the angle α. From the centre O draw O T, meeting A B produced in the point T, and in O T take the point Q, such that T Q = B T. Draw B O′, T O′ perpendicular to A B and T Q respectively intersecting each other in the point O′. With centre O′ and radius O′ B = O′ Q describe the arc B Q. If the vane be so designed that B Q represents the absolute path, the conditions laid down will manifestly have been complied with, since the water moves initially tangentially to the absolute path, and finally radially at right angles to the direction of motion. Whilst in outward-flow turbines the radius vector of the absolute path always lies between the tangent and a line parallel to the initial radius through the whole length of the absolute path, in inward-flow turbines the line parallel to the initial radius lies between the radius vector and the tangent to the absolute path up to the point where that tangent becomes parallel to the initial radius vector, and afterwards the tangent lies between the radius vector and the parallel line, therefore the angle between the tangent and the initial radius will be equal to $\phi - \theta$ up to that point, and afterwards to $\theta - \phi$.

If, therefore, $v_1 \sin a$ remained constant, leaving friction and curvature of absolute path out of consideration, up to the point where $\theta = \phi$, the radial velocity at any point between B and that point would be equal to $v_1 \sin a \sec (\phi - \theta) \cos \phi$, which is always less than $v_1 \sin a \sec \theta$, since $\cos \theta \cos \phi$ is less than $\cos (\phi - \theta)$. Hence it is evident that up to this point the vane cannot be designed on the supposition that the radial velocity is constant and equal to $v_1 \sin a$, since it will be still further diminished by friction. At this point the absolute velocity is parallel and equal to $v_1 \sin \alpha$.

After passing this point, the absolute velocity must decrease, and therefore always be less than $v_1 \sin a$, so that the velocity parallel to the initial radius must decrease at a much greater rate, since it is equal to the absolute velocity multiplied by $\cos (\theta - \phi)$. Since no loss of velocity parallel to the initial radius is assumed to accrue from friction or curvature of absolute path, it follows that the head due to the velocity of this component lost after the absolute velocity has become equal to $v_1 \sin a$ must have been expended in doing work, and there will be a slight increase in the efficiency of the machine due to the head so lost.

At the point, therefore, where the absolute is parallel to the initial radial velocity, we may look upon the jet moving along the absolute path as a jet striking the vane in a direction inclined to the direction of motion at an angle $\frac{\pi}{2} - \theta$. If, therefore, we resolve the absolute velocity in two directions, radial and tangential, we shall have radial velocity $v_1 \sin a \cos \theta$, and tangential equal to $v_1 \sin a \sin \theta$. Up to the point where the absolute velocity becomes parallel and equal to the initial radial velocity $v_1 \sin a \cos \theta$, the radial velocity will be less than the initial radial velocity, just as in the first part of the path and the tangential velocity $v_1 \sin a \sin \theta$ having been wholly destroyed, there will have been effective work done in proportion to the head lost, viz., $\frac{v_1^2 \sin^2 a \sin^2 \theta}{2g}$. Similarly, if θ' be the inclination of the radius vector at this point to the initial radius O B, we may resolve the absolute velocity $v_1 \sin a \cos \theta$ into two components

$v_1 \sin \alpha \cos \theta \cos (\theta' - \theta)$ radial and $v_1 \sin \alpha \cos \theta \sin (\theta' - \theta)$ tangential and so on *ad infinitum.*

Thus we see that, whereas in outward-flow turbines the only difficulty we have to contend with is the securing of proper consecutive sectional areas in the passages between the guide blades, a difficulty easily overcome, in inward-flow turbines the difficulty to be overcome is the determination of the proper form of the vane. The following consideration will, however, much simplify the problem we have to solve. At the point where the absolute velocity becomes parallel and equal to the component $v_1 \sin \alpha$, the angle θ will always be less than 45° whatever may be the value of the ratio $R_0 \div R_1$, and therefore the head lost $\frac{v_1^2 \sin \alpha \sin^2 \theta}{2g}$ will never be greater than $\frac{v_1^2 \sin^2 \alpha \sin^2 \frac{\pi}{4}}{2g}$, which for $\alpha = 20°$, a maximum value is less than one per cent. of the whole head, therefore we may neglect the effect of the jet during the second and succeeding stages without affecting in any appreciable degree the total efficiency, and the vane must be constructed as follows.

Through O′ draw O′ S parallel to C B, the initial tangent at B cutting the radius O B in *b*, divide the space B *b* into a succession of equal intervals *t*, *t′*. Draw *t p*, *t′ p′* at right angles to O B, meeting the absolute path in the points *p*, *p′*. Describe the arcs *p* P, *p′* P′, &c. Then if the lengths P *p*, P′ *p′* be so determined that the vane sweeps on the arcs P *p*, P′ *p′*, &c., whilst the jet moves parallel to the initial radius through the spaces B *t*, B *t′*, &c., it is evident that the locus of the points P, P′, &c., will represent the correct shape of the vane corresponding to the absolute path B Q. Since it is impossible to ascertain either the total loss of velocity parallel to B O or the rate of loss, we can only design the vane on the supposition that the component of the velocity parallel to B O, the initial radius remains constant. The best angular velocity will be therefore somewhat less than the angular velocity on which the calculations are based, and must be determined subsequently by actual experiment. Since $Bt = OB - Ot = R_1 - \rho \cos \theta$, the equation of relation between the successive arcs P *p*, &c., and the corresponding radii vectores through P will be

$$P p = \frac{(R_1 - \rho \cos \theta)\, \omega \rho}{v_1 \sin \alpha}.$$

Whilst in the case of outward-flow turbines, the arcual interval between consecutive vanes increases from the receiving towards the discharging side, and the areas increase more rapidly than is necessary to compensate for frictional loss of radial velocity, which also, leaving friction out of consideration, increases from the receiving towards the discharging side; in the case of inward-flow turbines, not only does the radial velocity decrease, but the areas at right angles to it, when the depth of the revolving drum parallel to the axis remains constant. It is evident, therefore, that the depth must increase from the receiving towards the discharging side. Since the area at right angles to the direction of motion at any point in the absolute path is equal to $\frac{2\pi\rho d}{n}$, leaving the thickness of the vane out of consideration, the projection of this area on a plane at right angles to its initial radius will lie between $\frac{2\pi\rho d \cos\theta}{n}$ and $\frac{2\pi\rho d \cos\theta'}{n}$, where θ' is the angle between the initial radius O B and the radius vector through the point where the arc radius ρ cuts the preceding vane. Now the angle θ increases from the value zero up to δ, whilst θ' decreases from the value $\frac{360°}{n}$ till it passes through the values zero and then begins to increase; till it equals zero; or finally reaches a minimum value according as $\frac{360°}{n}$ is less, equal to, or greater than δ. Up to the point, then, where $\theta = \theta'$, the depths must be chosen so that

$$d = \frac{D_1 R_1 \sec\theta'}{\rho},$$

and after this point

$$d = \frac{D_1 R_1 \sec\theta}{\rho}.$$

The areas will be somewhat larger than is necessary, if $v_1 \sin a$ remain constant, and therefore some allowance will have been made for friction. The simplest way, and one equally efficient, would be to make the whole depth constant and equal to $\frac{R_1 D_1 \sec\delta}{R_0}$, D_1 in this case being the depth of the guide-blade chamber.

Another artifice adopted to overcome this difficulty is to make every alternate vane half the full length. Since the actual increase of area, however, is small, and only equal to the tangential component of the thickness of the vane, it is evident that this expedient to prevent loss of initial velocity can only succeed by admitting of a greater final absolute velocity of discharge, and must therefore be accompanied by a diminution of efficiency.

Since the issuing water leaves the turbine parallel to the axis, the internal diameter of the revolving drum must be determined in the same way as the internal diameter of the guide-blade chamber in an outward-flow turbine. Since, however, the water escapes both ways, upwards and downwards, it is evident that the annular area need only be half the magnitude, or the radius of the discharging side of an inward-flow turbine need not exceed the radius of the receiving side of the guide-blade chamber of an outward-flow turbine divided by $\sqrt{2}$.

After escaping from this annular space, the issuing water flows over and under the turbine casing. Hence, the greater the height of the surface of the tail water above the top of the turbine the less will be the loss of head due to the velocity of the tail water. It is evident also that the area of the tail water below the turbine ought to be equal to the area above it, and the sum of the two to the whole area of the tail water in the case of an outward-flow turbine, so that the depth of the tail water in the former case in the turbine pit itself must exceed that in the latter by the depth of the casing of the inward-flow turbine.

The elements of the absolute path are determined as follows:

In Fig. 8, since B T, T Q, are the tangents to the absolute path at B and Q, we must have B T = Q T, also

$$\mathrm{B\,T} = \frac{\mathrm{B\,O}\sin \mathrm{B\,O\,T}}{\sin \mathrm{B\,T\,O}} = \frac{R_1 \sin\delta}{\cos(\alpha-\delta)}$$

$$\mathrm{T\,Q} = \mathrm{T\,O} - \mathrm{O\,Q} = \frac{\mathrm{B\,O}\sin \mathrm{O\,B\,T}}{\sin \mathrm{B\,T\,O}} - \mathrm{O\,Q}$$

$$= \frac{R_1\cos\alpha}{\cos(\alpha-\delta)} - R_0$$

$$R_1(\cos\alpha - \sin\delta) = R_0\cos(\alpha-\delta)$$

putting $\delta = \frac{\pi}{2} - \phi$, and reducing we get

$$\tan\frac{\phi}{2} = \frac{R_1\tan\frac{\alpha}{2} + R_0}{R_1 + R_0\tan\frac{\alpha}{2}}$$

$$\tan\frac{\delta}{2} = \frac{\left(R_1 - R_0\right)\left(1 - \tan\frac{\alpha}{2}\right)}{\left(R_1 + R_0\right)\left(+\tan\frac{\alpha}{2}\right)}$$

also

$$O'B = BT \tan \frac{BTO}{2}$$

$$= \frac{OB \sin BOT \tan \frac{BTO}{2}}{\sin BTO}$$

or

$$r = \frac{R_1 \sin \delta}{1 - \sin (\alpha - \delta)}.$$

The radius O Q′ of the cylinder, which intersects the vanes at the point where the absolute velocity is parallel to the initial radial velocity, is thus determined. Let it be designated by the symbol R'_0, then

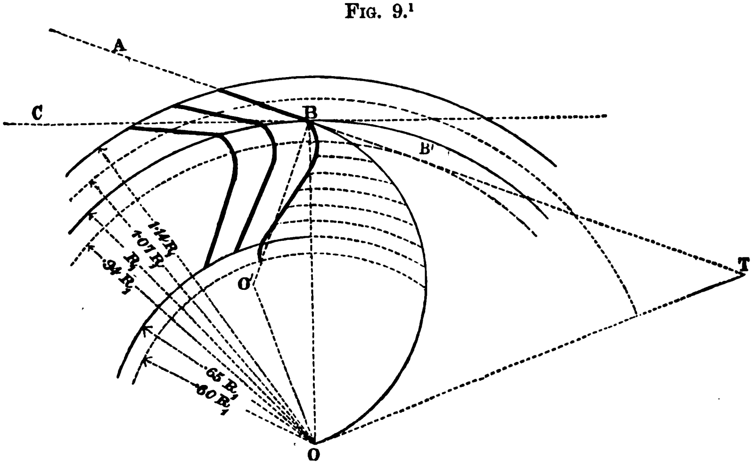
$$R_0' = OQ' = \sqrt{Ob^2 + bS^2}$$

$$Ob = R_1 - r \cos \alpha$$

$$bS = r - r \sin \alpha$$

$$R'_0 = \sqrt{R_1^2 + 2r^2 - 2r(R_1 \cos \alpha + r \sin \alpha)}$$

Fig. 9.[1]



The arrangement for the guide blades in inward-flow turbines is exceedingly simple. The very best possible form consists of straight vanes cutting the external circumference of the revolving drum at the constant angle α. These guide blades will all be tangents to a circle centre O and radius equal to $R_1 \cos \alpha$. Let the guide blade through B meet the circumference of the circle

[1] For explanation of Fig. 9 see pp. 63 and 64.

radius $R_1 \cos a$ in B′. Set off from B and B′ equal angular intervals on their respective circumferences, each subtending the angle $\frac{360^\circ}{n}$ at the centre. Lines passing through each successive pair of points B B′, $B_1 B'_1$, &c., will give the proper direction for the guide blades.

As in the case of outward-flow turbines, each guide blade must be at least sufficiently long to cut the normal from O through the point where the preceding guide blade cuts the inner periphery of the guide-blade chamber. This, however, would not be sufficiently long to give the proper direction to the jet, unless the direction of the approaching water were made to converge towards parallelism with that of the jet. The radius of the circumference in which the successive guide blades meet the above normals is equal to $R_1 \cos a \sec \left(\frac{360^\circ}{n} + a\right)$, because the angle between the radius through the point where a guide blade meets the circle radius $R_1 \cos a$, and the radius through the point, where the preceding guide blade cuts the circle radius R_1 is equal to a° plus the angle subtended by one of the openings. Since the turbine is wholly buried under the tail race, and the water issues both above and below the turbine, the plane of the direction of the velocity of approach must always be at right angles to the axis. If, in addition, that direction is also radial, the radius of the external circumference of the guide-blade chamber must be greater than $R_1 \cos a \sec \left(\frac{360^\circ}{n} + a\right)$. Inward-flow turbines have, however, an advantage over outward-flow turbines in this respect. The supply chamber may be so arranged that the direction of the velocity of approach gradually converges to parallelism with the direction of the jet, viz., by making the top and bottom flush with the top and bottom of the guide-blade chamber, and the other parallel to the axis, and gradually converging towards the external circumference of the guide-blade chamber till it ultimately meets a guide blade tangentially.

Fig. 10 is a horizontal section through such a turbine with the vanes omitted. The external casing is a spiral, determined by the condition that the interval between the casing and the supply chamber measured along a radius vector shall be to the fixed initial interval in the ratio of the number of intervening openings in the guide-blade chamber to unity. Produce the radius O B *b*, which passes through the points B, *b*, where two successive guide blades severally meet the discharging and receiving side of the guide-blade chamber, till it meets the next guide blade but one likewise produced in b'. Then $b\,b'$ will be a suitable initial interval. The radius O b' is equal to $R_1 \cos \alpha \sec\left(\frac{720}{n} + \alpha\right)$; therefore $b\,b' = R_1 \cos \alpha \left\{\sec\left(\frac{720}{n} + \alpha\right) - \sec\left(\frac{360°}{n} + \alpha\right)\right\}$. Now the equation to the spiral must be

$$\rho = a\,\theta,$$

and the value of a must be so determined, that, for a certain value of θ

$$R_1 \cos \alpha \sec\left(\frac{720}{n} + \alpha\right) = a\,\theta$$

and

$$R_1 \cos \alpha \left\{(n+1) \sec\left(\frac{720}{n} + \alpha\right) - n \sec\left(\frac{360}{n} + \alpha\right)\right\} = a\,(\theta + 2\,\pi);$$

whence the value of a is

$$\frac{n\,R_1 \cos \alpha \left\{\sec\left(\frac{720}{n} + \alpha\right) - \sec\left(\frac{360}{n} + \alpha\right)\right\}}{2\,\pi}$$

and the value of θ corresponding to the radius vector O B $b\,b'$ is

$$\frac{2\,\pi \sec\left(\frac{720}{n} + \alpha\right)}{n \sec\left(\frac{720}{n} + \alpha\right) - n \sec\left(\frac{360}{n} + \alpha\right)}$$

The tangent of the angle between the radius vector and the tangent to the spiral at any point is

$$\rho\,\frac{d\,\theta}{d\,\rho} = \frac{\rho}{a} = \theta,$$

or the tangent is equal to the circular measure of the angle swept out by the radius vector; therefore the spiral, whose equation is given above, will not meet the guide blade tangentially. The two must be joined by a short circular curve, and the next guide blade must be diminished in length to make up for this diminution on the opening $b\,b'$. The spiral might have been designed so as to meet this guide blade tangentially, but the increase of the sectional areas would be too great to admit of the use of such a spiral.

Since the angle between the radius vector and the tangent of the spiral increases with θ, the areas also at right angles to direction of the water increase more rapidly than in the ratio of the

E 2

number of intervening guide blades to unity, so that the velocity in the supply chamber gradually increases from the penstock up to the last orifice in the guide-blade chamber, by this means insuring the same value of the initial velocity v_1 at every office.

Through the point p where the first guide blade $b'p$ meets the circle radius $R_1 \cos a$ draw the radius vector O p P, meeting the spiral in P. Draw P N tangentially to the spiral. From the point M in the guide blade $p\,b'$ produced, draw M N parallel to O P and meeting P N in N.

A vertical section through M N will form a suitable orifice to the supply chamber.

Fig. 10.

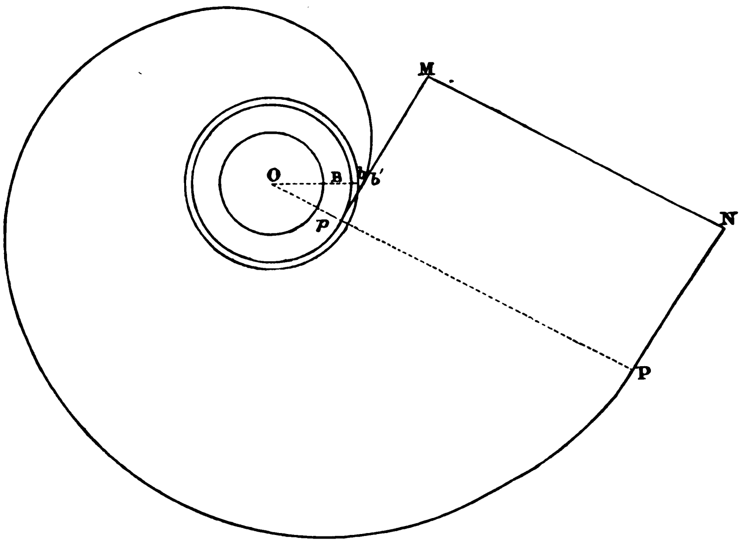


Fig. 11 represents the development of part of a section of a parallel-flow turbine made by a cylinder concentric with the axis. In this case the component of the velocity at right angles to the direction of motion at any point of the absolute path is always parallel to the initial component in the same direction, that is parallel to the axis of the turbine, consequently the tangent to the absolute path at the discharging side must be parallel to the axis.

Let Q be the point of discharge. Draw Q T parallel to the axis meeting A B, the tangent to the guide blade produced in T. Then the depth must be determined by the condition

$$QT = BT.$$

Draw B O, Q O at right angles to B T, Q T, respectively, then will

O be the centre of the absolute path. Draw B D at right angles to Q O meeting Q O in D, then will B D represent the depth D of the turbine, and we shall have

$$\text{O B} = \text{D B} \sec \text{O B D}$$

or

$$r = \text{D} \sec \alpha;$$

also if ρ be the radius of the intersecting cylinder, and δ the circular measure of the angle between two vertical planes through the axis and the points B and Q, we shall have

$$\delta = \frac{\text{Q D}}{\rho} = \frac{r - r \sin \alpha}{\rho}$$

$$= \frac{(1 - \sin \alpha) \text{D}}{\rho \cos \alpha}.$$

Unless, therefore, the ratio $\text{D} \div \rho$ be constant, or the depth increase from the inner towards the outer circumference, the angle δ will not be constant, and if D varies r will also vary, and consequently the curve of the absolute path will vary.

In the curve to the absolute path B Q, take successive points p, p' at equal intervals, and draw $p\, b$ P, $p'\, b'$ P$'$ parallel to the direction of motion, meeting B D in $b\, b'$ and the relative path in P, P$'$ respectively. If then the initial velocity, $v_1 \sin \alpha$, remain constant we must have

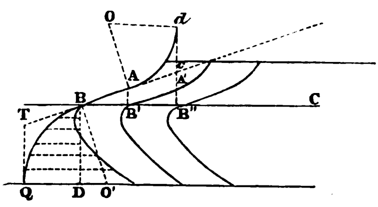
$$p\,\text{P} = \frac{\text{B}\, b\, \omega\, \rho}{v_1 \sin \alpha}$$

$$= \frac{d\, \omega\, \rho}{v_1 \sin \alpha},$$

where d is the depth from the receiving side of corresponding points in the absolute and relative path.

Now the value of ρ varies from R_1 to R_0, therefore theoretically, whether D, and consequently the radius of the absolute path, remain constant or not, the section of the vane ought to vary accordingly. In parallel-flow turbines, then, we have two difficulties to contend with—the impossibility of ascertaining the loss due to

FIG. 11.



friction of the velocity parallel to the axis, and the complication due to the varying value of the radius. If, however, the vane be designed to suit the extreme section R_0, the length of time it takes at any other section to describe the interval $\omega (R_0 - \rho)$ will compensate, and may more than compensate, as ρ approaches the value R_1 for the loss in the component $v_1 \sin \alpha$ due to friction; so that for some intermediate value of ρ the shape of the vane will be such as to cause the water to move in the assigned absolute path. Owing likewise to this variation in the value of the radius the ratio velocity of vane $\div$ component $v_1 \cos \alpha$ of the velocity of the jet varies unless the value of α varies also; but if α varies so as to make ratio $\omega \rho \div v_1 \cos \alpha$ constant, the component parallel to the axis will vary; so that we cannot by varying the angle α design a vane of uniform section whose initial tangent shall at every section be parallel to the relative path. The only way to get over this difficulty is to make the value of the ratio $R_0 \div R_1$ so small that this variation will not appreciably affect the efficiency. It has been shown in Part I. that a variation of 10 per cent. either way, leaving curvature of vane out of consideration, does not materially diminish the efficiency, and therefore we ought to have

$$\frac{R_1 + R_0}{2} \text{ not less than } R_0 - \frac{R_0}{10}$$

or

$$R_0 \text{ not greater than } 1 \cdot 25\, R_1.$$

Since the initial velocity $v_1 \sin \alpha$, parallel to the axis, diminishes by friction, it is evident that the successive areas made by planes at right angles to the axis ought to increase from the receiving towards the discharging side. This may be done by gradually increasing the difference between R_0 and R_1, or more simply by making the width of the guide-blade chamber less than that of the vane chamber.

The form of the guide blades is very simple. Let lines parallel to the axis through consecutive points B′, B″, &c., where the guide blades meet the receiving side of the revolving drum cut the preceding straight portion of the guide blade in the points A, A′, &c. Produce B″ A′ till it meets B A in c, and in B″ c produced take the point d such that $d\,c = A\,c$. Through d draw d O parallel to B C, meeting the perpendicular from A on B A in O. With centre O and radius O A describe the arc A d. Then will B A d be a suitable shape for the guide blade. The depth of the guide-blade chamber is equal to

$$\begin{aligned} d\,c + c\,B'' &= A\,c + c\,B'' \\ &= B\,B' (\sec \alpha + 2 \tan \alpha) \\ &= \frac{2\,\pi\,\rho\,(\sec \alpha + 2 \tan \alpha)}{n}. \end{aligned}$$

In theory, therefore, the guide-blade chamber ought to vary in depth to suit the varying value of ρ. So long, however, as the successive points B′, B″, &c., lie between the successive points B, B′, &c., and the verticals through the successive points A, A′, &c., the jet will issue with its proper direction. This condition will be satisfied if we make the guide blades suit the external section of the guide-blade chamber. The great defect in parallel-flow turbines is that the vertical planes parallel to the direction of the jet do not cut the vanes parallel to the direction of motion, so that the actual absolute path does not coincide with B Q, but corresponds with the intersection of the vane made by a vertical plane tangential to the direction of motion at the point B, and is therefore elliptic. There will therefore be, in addition to the final velocity $v_1 \sin \alpha_1$, leaving friction out of consideration, a small component of velocity at right angles to the direction of motion in a horizontal plane at the point Q.

We have now to consider the best values of the ratio, $R_0 \div R_1$ in outward and inward-flow turbines and of the depth D in parallel-flow turbines.

We may consider the question in two ways—

1st. What must this relation be in order that the whole head lost in passing through the turbine may be a maximum.

2nd. What must this relation be in order that the ratio, power developed divided by the power lost, may be a maximum.

In outward-flow turbines, constructed in accordance with the rules previously laid down, the final absolute velocity, leaving friction out of consideration, is equal to $v_1 \sin \alpha \sec \delta$, and therefore the head lost is equal to $\frac{v_1^2 (1 - \sin^2\alpha \sec^2\delta)}{2g}$, and therefore the less δ the greater the whole head lost. The variation in the value of $v_1 \sin \alpha \sec \delta$ for all values of δ less than α is so small that the head lost will not be affected by making the value of μ less than that due to this value of δ which is given by the equation

$$\tan \frac{\delta}{2} = \frac{\left(\mu - 1\right)\left(1 - \tan \frac{\alpha}{2}\right)}{\left(\mu + 1\right)\left(1 + \tan \frac{\alpha}{2}\right)}$$

or

$$\mu = \frac{1 + \tan^2 \frac{\alpha}{2}}{1 - 2 \tan \frac{\alpha}{2} - \tan^2 \frac{\alpha}{2}}.$$

If $a = 20°$ the head lost in passing through the turbine will be $\frac{\cdot 87\, v_1^2}{2\, g}$ and the value of μ 1·67. If $a = 10°$ the head lost will be $\frac{\cdot 96\, v_1^2}{2\, g}$ and the value of μ 1·24. Since sin a = tan a very nearly, it is evident that no appreciable increase of efficiency could have arisen from making the final tangent to the absolute path parallel to the direction of the initial radials component v_1 sin a, whilst the solution would have been complicated by the radial velocity becoming, as in the case of inward-flow turbines, less than v_1 sin a.

In inward-flow turbines, constructed in accordance with the principles already laid down, the final absolute velocity, leaving friction out of consideration, is equal to v_1 sin a, and therefore the head lost to $\frac{v_1^2 \cos^2 a}{2\, g}$, which is the same for all values of the ratio. If, therefore, in this case the value of δ be equal to a, since the same equation of relation exists between a and δ, the head lost, if $a = 20°$ will be $\frac{\cdot 88\, v_1^2}{2\, g}$ and 1·67 the value of μ. If $a = 10°$ the head lost will be $\frac{\cdot 97\, v_1^2}{2\, g}$ and 1·24 the value of μ.

In inward-flow turbines, since the absolute velocity of the parts of the vane which are nearer the centre are less than those more remote, if the vanes are so designed that the absolute path passes through the centre of the axis, there would be many changes of curvature in the relative path, the points of contrary flexure corresponding with those of the absolute path at which the absolute velocity is parallel to the successive initial radii already described. It will not, however, add appreciably to the efficiency to carry the vane beyond the *first* point of contrary flexure. This point will be further discussed during the investigation of the value of the ratio $R_1 \div R_0$, which makes the ratio effective power $\div$ power lost a maximum.

In parallel-flow turbines vertical planes parallel to the direction of motion diverge from parallelism with the corresponding initial plane, and the absolute velocity may be resolved into three components: 1st, parallel to the axis, which is supposed to remain constant and equal to v_1 sin a. 2nd, parallel to the direction of motion. 3rd, radial. The head due to this last component is wholly lost. Since the angle between the final and initial tangent planes is equal to the angle δ between the radii passing through

B and Q, it is evident that the smaller this angle is the less will be the loss of head due to this cause. Now, since δ is small, we have

$$\tan \delta = \frac{\text{arc DQ}}{\text{radius}}$$

$$= \frac{D(\sec a - \tan a)}{\rho}$$

or

$$D = \frac{\rho \tan \delta \cos a}{1 - \sin a}.$$

If $a = 20°$ and $\delta = 10°$ we shall have

$$D = \cdot 25\,\rho.$$

If $a = 10°$ and $\delta = 10°$ we shall have

$$D = \cdot 2\,\rho.$$

Therefore, theoretically, the depth ought to increase from the inner to the outer circumference of the vane chamber. All the conditions will, however, be satisfied if the depth remain constant and equal to $\cdot 25\ R_1$ or $\cdot 2\ R_1$ according as a is equal to 20° or 10°.

We now come to the next point, viz., what value of the ratio μ must be adopted in order that the ratio effective power $\div$ power lost may be a maximum.

Let (Fig. 7) p, p' be contiguous points in the curve of the absolute path of an outward-flow turbine at the distances ρ and $\rho + \delta\rho$ from the axis respectively. Draw the arcs p P, p' P′ meeting the relative path in the points P and P′ respectively. Produce Op till it meets the arc p' P′ in r and O P till it meets p' P′ or p' P′ produced in r'. From r along the arc p' P′ measure off $r\,q$ equal to r' P′, the point q being remote from or adjacent to the point p' according as O P produced cuts p' P′ or p' P′ produced in r'. Then the angle $q\,p\,r$ is evidently equal to the angle P′ P r', which the radius vector O P makes with the chord P P′ of the relative path. When $\delta\rho$ is indefinitely diminished, this angle is equal to the angle between the tangent at P and the radius vector O P. Similarly the angle $r\,p\,p'$ is ultimately equal to the angle between the radius vector Op and the tangent to the absolute path at p. Therefore the angle $q\,p\,p'$ will ultimately be equal to the complement of the angle between the direction of the motion of the water at the point p and the normal to the vane when the point P of the relative path coincides with the point p of the absolute path. Hence if we know the velocity of the water and that of the element P P′ of the vane at right angles to the chord P P′, we can ascertain the normal moving force exerted per second on this element of the vane, and therefore its component parallel to the direction of motion, which, multiplied by the velocity of the point P will give the effective work done on this element, when $\delta\,\rho$ is indefinitely diminished, and the sum of these acting at each point of the vane will represent the whole work per second done by the water in passing through the turbine. Let

$$\angle\, q\,p\,r \text{ be denoted by } \psi'$$
$$\angle\, r\,p\,p' \quad \text{,,} \quad \text{,,} \quad \phi'$$

then the complement of the angle between the direction of the water and the normal to the vane will be equal to $\phi' \pm \psi'$ according as O P meets p' P′ or p' P′ produced in r'.

Since the radial velocity is assumed to be constant and equal to the initial radial velocity $v_1 \sin a$, the absolute velocity at any point will be equal to

$$v_1 \sin a \sec \phi,$$

where ϕ is the angle between the radius vector and the tangent to the absolute path. Also the angle between the normal to the chord P P′ and the direction of whirl is equal to the angle between the chord and the radius vector $= r'$ P P′ $= q\,p\,r = \psi'$, and therefore the normal moving force exerted on the element P P′ per second will be equal to

$$\frac{\mathrm{W}}{g}\left\{ v_1 \sin a \sec \phi \sin (\phi' \pm \psi') - \omega\,\rho \cos \psi' \right\}$$

where W is the weight of water discharged per second. This multiplied by $\cos \psi'$ will be equal to the moving force exerted in direction of motion, and therefore the work done per second in direction of motion will be equal to

$$\frac{\mathrm{W}}{g}\left\{ v_1 \sin a \sec \phi \sin (\phi' \pm \psi') - \omega\,\rho \cos \psi' \right\}\omega\,\rho \cos \psi',$$

and the summation of these elements from the value R_1 to R_0 of ρ will give the total work done per second by the water. In the case of the absolute path, the acute angle between the tangent and the radius vector never passes through the value zero until it attains its ultimate value. The ultimate value of the angle ϕ' is therefore equal to ϕ. On the contrary, in the case of the relative path, the angle between the tangent and the radius vector does pass through the value zero, and after passing through the value zero, since it is always measured on the same side of the tangent, this angle will be equal to the supplement of the acute angle, which the radius vector makes with the tangent. This acute angle is in all cases equal to the ultimate value of ψ'. If, therefore, ψ be the angle between the tangent to the relative path and the radius vector, we shall have

$$\sin(\phi' - \psi') = \sin(\phi - \psi)$$
$$\sin(\phi' + \psi') = \sin(\phi + 180^\circ - \psi) = -\sin(\phi - \psi);$$

$\sin(\phi - \psi)$ being negative when q is remote from p'.

Although the ultimate value of ϕ' and ψ' are equal respectively to ϕ and ψ, or $180^\circ - \psi$, we cannot substitute these values in the expression for the work done per second on any element, since this would simply make it vanish.

The value above given has been investigated for the sake of illustrating the exact theoretical principles by means of which the problem must be solved. We may arrive at an approximate solution by the following artifice.

Let ϕ, ϕ' and ψ, ψ' be the angles between the radii vectores and the tangents to the absolute and relative paths respectively, corresponding to the values ρ and $\rho + \delta\rho$ of the radii vectores. The total absolute velocity at each point will be equal to $v_1 \sin a \sec \phi$, and $v_1 \sin a \sec \phi'$ respectively, and when $\delta\rho$ is indefinitely diminished, we may consider that the angle between the tangents to the absolute and relative paths at the point, whose radius vector is $\rho + \delta\rho$, is equal to the corresponding angle at the point whose radius vector is ρ, or to $\phi - \psi$, and therefore the velocity normal to the relative path at the two instants will be equal to $v_1 \sin a \sec \phi \sin(\phi - \psi)$ and $v_1 \sin a \sec \phi' \sin(\phi - \psi)$ respectively, and the difference between these, viz.,

$$\pm\, v_1 \sin a \sin(\phi - \psi)\,\{\sec\phi - \sec\phi'\}$$

will represent the velocity destroyed normally to the vane, which is the only component that has any effect upon the motion, the positive sign denoting a motion towards, the negative from the centre. Therefore the work done in direction of motion will be equal to

$$\frac{W}{g}\, v_1 \sin a \sin(\phi - \psi)\,\{\sec\phi - \sec\phi'\}\,\omega\rho\cos\psi,$$

$\cos\psi$ being negative, when negative sign comes before $\sin(\phi - \psi)$. If we take O for origin and O O′ for initial line, and denote the angle O′ O p by θ, we shall have

$$OO'^2 = O'Q^2 + OQ^2 = r^2 + R_0^2$$

and

$$O'p^2 = OO'^2 + Op^2 - 2OO'.Op\cos\theta;$$

therefore the equation to the absolute path will be

$$\rho^2 - 2\rho\sqrt{r^2 + R_0^2}.\cos\theta + R_0^2 = O;$$

also if θ' be equal to the angle O′OP, we shall have the arc $p\,\mathrm{P} = \rho(\theta - \theta')$. Since $p\,\mathrm{P}$ is also equal to $\frac{\omega\rho(\rho - \mathrm{R}_1)}{v_1 \sin a}$, we get the equation of relation:

$$\theta - \theta' = \frac{\omega(\rho - \mathrm{R}_1)}{v_1 \sin a},$$

also

$$\tan\phi = \rho\frac{d\theta}{d\rho} = \frac{\sqrt{r^2 + \mathrm{R}_0^2}\cos\theta - \rho}{\sqrt{r^2 + \mathrm{R}_0^2}\sin\theta}$$

$$= \frac{\mathrm{R}_0^2 - \rho^2}{\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}},$$

$$\tan\psi = \rho\frac{d\theta'}{d\rho} = \frac{\mathrm{R}_0^2 - \rho^2}{\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}} - \frac{\omega\rho}{v_1 \sin a};$$

therefore

$$\sec\phi = \sqrt{1 + \tan^2\phi} = \frac{2\rho r}{\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}},$$

also

$$\sec\phi' = \frac{2(\rho + \delta\rho)r}{\sqrt{4(\rho + \delta\rho)^2 r^2 - \{\mathrm{R}_0^2 - (\rho + \delta\rho)^2\}^2}};$$

therefore approximately, having regard to the double sine, of the root:

$$\sec\phi - \sec\phi' = \frac{2r\,d\rho}{\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}}.$$

Therefore the whole work done per second will be equal to

$$\frac{2\,\mathrm{W}\,v_1\,r\sin a}{g}\int_{\mathrm{R}_1}^{\mathrm{R}_0}\frac{\sin(\phi - \psi)\,\omega\rho\cos\psi\,d\rho}{\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}}.$$

Now

$$\sin\phi = \frac{\tan\phi}{\sqrt{1 + \tan^2\phi}} = \frac{\mathrm{R}_0^2 - \rho^2}{2\rho r};$$

similarly,

$$\sin\psi = \frac{(\mathrm{R}_0^2 - \rho^2)v_1\sin a - \omega\rho\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}}{\sqrt{u}},$$

where

$$u = 4\rho^2 r^2(v_1^2\sin^2 a + \omega^2\rho^2) - \omega^2\rho^2(\mathrm{R}_0^2 - \rho^2)^2 - 2v_1\sin a\,.\,\omega\rho(\mathrm{R}_0^2 - \rho^2) \times \sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}$$

$$\cos\psi = \frac{v_1\sin a\sqrt{4\rho^2 r^2 - (\mathrm{R}_0^2 - \rho^2)^2}}{\sqrt{u}}.$$

Therefore

$$\sin(\phi - \psi) = \frac{\omega \{4\rho^2 r^2 - (R_0^2 - \rho^2)^2\}}{2 r \sqrt{u}}.$$

Therefore the whole work done will be equal to

$$\frac{W \omega^2 v_1^2 \sin \alpha}{g} \int_{R'}^{R_0} \frac{\rho \{4\rho^2 r^2 - (R_0^2 - \rho^2)^2\} d\rho}{u}.$$

The expression just found is not integrable. If we examine the values of the separate terms in the numerator and denominator, we see that for all values of ρ less than R_0 the value of the numerator increases as ρ increases. In the denominator the positive term constantly increases as ρ increases, whatever may be the value of ρ. The first negative term

$$\omega^2 \rho^2 (R_0^2 - \rho^2)^2$$

is a maximum when $\rho = \frac{R_0}{\sqrt{3}}$. The second negative term contains the two variable factors

$$\rho (R_0^2 - \rho^2) \text{ and } \sqrt{4\rho^2 r^2 - (R_0^2 - \rho^2)^2},$$

of which the first has a maximum value, when $\rho = \frac{R_0}{\sqrt{3}}$, and the second when $\rho = R_0$. Therefore the value of ρ which gives a maximum value to the negative terms lies between R_0 and $\frac{R_0}{\sqrt{3}}$, or we must always have $R_0 \div R_1$ not less than 1·73. We may therefore fix upon the value of $\mu = 1 \cdot 8$ as a minimum one for determining the elements of the absolute path.

Since, however, the absolute velocity towards the end of the absolute path is very nearly parallel to the radius, and the tangents to the relative path inclined at very small angles to the direction of motion, the effective work done towards the end of the absolute path will be of very little moment, whilst the resistance due to the friction of the water varies as the square of the external radius approximately. The external radius of the guide-blade chamber ought therefore to be less than the radius R_0 used for determining the elements of the absolute path.

Let $c v_1$ be the absolute velocity corresponding to the value ρ of the radius vector, then

$$c v_1 = \frac{2 \rho r v_1 \sin \alpha}{\sqrt{4\rho^2 r^2 - (R_0^2 - \rho^2)^2}},$$

whence

$$\rho^4 - 2\,\rho^2 \left\{ R_0^2 + 2\,r^2 \left(1 - \frac{\sin^2 a}{c^2} \right) \right\} + R_0^4 = 0.$$

If we substitute for r, R_0, their values in terms of R_1, *i.e.*, putting $R_0 \div R_1 = 1 \cdot 8$, we shall obtain a relation between ρ and R_1 in terms of c, to which we may assign any value we choose.

If $a = 20°$, we get $\delta = 22° \; 37' \; 44''$ $r = 1 \cdot 192\ R_1$, and if $c = \cdot 5$, so that the head expended in passing through the turbine is equal to $\frac{\cdot 75\, v_1^2}{2\, g_1}$, we shall have outside radius of turbine case equal to $1 \cdot 5\ R_1$ nearly.

In the case of inward-flow turbines, the radial velocity being always less than $v_1 \sin a$, leaving friction out of consideration, the assumption that they are equal is not so nearly true as in the case of outward-flow turbines. It will, however, be sufficiently exact to determine approximately the best value of the ratio $R_1 \div R_0$. On this assumption, we arrive at exactly the same expression for the amount of work done in passing over any element of the path, viz.:

$$\frac{W}{g}\, v_1 \sin a \sin(\phi - \psi)(\sec \phi - \sec \phi')\, \omega\, \rho \cos \psi.$$

The angle ψ, as we have already seen, may pass more than once through the value zero.

The equation to the absolute path referred to origin O and initial line O O′ will be equal to

$$\rho^2 - 2\,\rho\,\sqrt{r^2 + R_0^2} \cos \theta + R_0^2 = 0;$$

therefore, writing $\rho^2 - R_0^2$ for $R_0^2 - \rho^2$, since ρ is greater than R_0,

$$\tan \phi = \rho \frac{d\,\theta}{d\,\rho} = - \frac{\rho^2 - R_0^2}{\sqrt{4\,\rho^2 r^2 - (\rho^2 - R_0^2)^2}}$$

$$\sec \phi = - \frac{2\,\rho\, r}{\sqrt{4\,\rho^2 r^2 - (\rho^2 - R_0^2)^2}}$$

$$\sin \phi = \frac{\rho^2 - R_0^2}{2\,\rho\, r};$$

the negative sign being used before the roots in the case of $\tan \phi$ and $\sec \phi$, and the positive before the root in the case of $\sin \phi$, because the two former are always negative and the latter always positive.

The equation of relation between θ and θ', the angle between the initial line and the radius vector to the relative path, is

$$\theta - \theta' = \frac{\omega\,(R_1 - \rho \cos \theta)}{v_1 \sin a};$$

therefore

$$\tan\psi = \rho\frac{d\theta'}{d\rho} = \rho\frac{d\theta}{d\rho} + \frac{\omega\rho\cos\theta}{v_1\sin a} - \frac{\omega\rho^2\sin\theta\frac{d\theta}{d\rho}}{v_1\sin a}$$

$$= \frac{\omega\rho^2}{v_1\sin a\sqrt{r^2+R_0^2}} - \frac{\rho^2 - R_0^2}{\sqrt{4\rho^2r^2-(\rho^2-R_0^2)^2}}$$

$$\sin\psi = \frac{\omega\rho^2\sqrt{4\rho^2r^2-(\rho^2-R_0^2)^2} - v_1\sin a(\rho^2-R^2)\sqrt{r^2+R_0^2}}{\rho\sqrt{u}}$$

$$\cos\psi = \frac{v_1\sin a\sqrt{r^2+R_0^2}\sqrt{4\rho^2r^2-(\rho^2-R_0^2)^2}}{\rho\sqrt{u}},$$

where

$$u = 4r^2\{\omega^2\rho^4 + v_1^2\sin^2 a(r^2+R_0^2)\} - \omega^2\rho^2(\rho^2-R_0^2)^2 - 2v_1\sin a\,.\,\omega(\rho^2-R_0^2)\sqrt{r^2+R_0^2}\sqrt{4\rho^2r^2-(\rho^2-R_0^2)^2}.$$

Therefore

$$\sin(\phi-\psi) = \frac{\omega\{4\rho^2r^2-(\rho^2-R_0^2)^2\}}{2r\sqrt{u}}$$

$$\sec\phi - \sec\phi' = \frac{2r\,d\rho}{\sqrt{4\rho^2r^2-(\rho^2-R_0^2)^2}},$$

and expression for the whole effective work done becomes equal to

$$\frac{W\omega^2v_1^2\sin^2 a\sqrt{r^2+R_0^2}}{g}\int_{R_0}^{R_1}\frac{\{4\rho^2r^2-(\rho^2-R_0^2)^2\}\,d\rho}{u}.$$

If we examine the terms in the denominator, we see that the values of the negative terms increase as ρ increases; we cannot, therefore, as in the case of outward-flow turbines, determine the best value of the ratio $R_1 \div R_0$ from an analysis of the value of the denominator.

The terms in the numerator, whose values depend upon that of the ratio $R_1 \div R_0$, are $\sqrt{r^2+R_0^2}$ and $4\rho^2r^2-(\rho^2-R_0^2)^2$ respectively. The maximum value of the second corresponds with the value $\rho = \sqrt{2r^2+R_0^2}$, and is equal to $4r^2(r^2+R_0^2$, and the product of the two is equal to $4r^2(r^2+R_0^2)^{\frac{3}{2}}$, the maximum value of which manifestly corresponds with the maximum value of r. Now r increases as R_0 decreases, and the value of r is a maximum when R_0 equals zero, and therefore the curve of the absolute path, when prolonged, ought to pass through the centre of the axis. Since in this case $R_0 = o$, we have

$$\tan\frac{\delta}{2} = \frac{1 - \tan\frac{a}{2}}{1 + \tan\frac{a}{2}}$$

also

$$r = \frac{R_1 \sec a}{2}$$

because a perpendicular from O′ on O B bisects O B.

$$\text{If } a = 20^\circ$$
$$\delta = 69^\circ\ 57'\ 50''$$
$$r = \cdot 532\ R_1$$

the value of ρ, which makes the numerator a maximum, is therefore equal to $\cdot 75\ R_1$, and the radius of the cylinder which passes through the point, where the absolute velocity is equal and parallel to $v_1 \sin a$, is equal to

$$\sqrt{\frac{R_1^{\,2}}{2} + r^2 (1 - \sin a)^2} = \cdot 6\ R_1.$$

The actual radial velocity at any point, leaving friction out of consideration, is equal to the component $v_1 \sin a$ multiplied by the cosine of the angle between the initial radius vector of the absolute path and the radius vector at the point $(\rho\,\theta)$, or to $v_1 \sin a \cos (\theta - a)$, and therefore the absolute velocity is equal to

$$v_1 \sin a \cos (\theta - a) \sec \phi = \left(\frac{\rho \cos a \sin a}{\sqrt{4 r^2 - \rho^2}} + \sin^2 a\right) v_1 = c \,.\, v_1.$$

If $a = 20^\circ$, $\rho = \cdot 8\ R_1$, when $c = \cdot 5$ and $\rho = \cdot 67\ R_1$ when $c = \cdot 4$.

In the case of parallel-flow turbines, take O′ for origin, and O′B for initial line. Draw the radii vectores O′ p, O′ P to the two corresponding points p P in the absolute and relative path. Then, since the equation of the absolute path is

$$\rho = r,$$

the angle ϕ is always equal to $\frac{\pi}{2}$; therefore the complement of the angle between the tangent to the absolute path and the normal to the relative path at any instant is equal to $90^\circ + \psi - (\theta - \theta')$, or $\psi - (\theta - \theta') - 90^\circ$, according as ψ is less or greater than 90°. Also the angle between a tangent to the absolute path and the direction of the component $v_1 \sin a$ at any point is equal to the complement of the angle between the latter and the radius vector through the point of the absolute path referred to, or to $\frac{\pi}{2} - (\theta + a)$, and the angle between the direction of motion and the normal through any point P of the vane is equal to the angle between the tangent to the vane and the direction of the component $v_1 \sin a$, or to $\psi + a + \theta$, or $180° - (\psi + a + \theta)$. Therefore the absolute velocity at any point will be equal to $v_1 \sin a \operatorname{cosec}(a + \theta)$, and the loss of velocity in passing to the adjacent point $(\rho, \theta + \delta\theta)$ will be

$$v_1 \sin a \{\operatorname{cosec}(a + \theta) - \operatorname{cosec}(a + \theta + \delta\theta)\} = \frac{v_1 \sin a \cos(a + \theta)\, \delta\theta}{\sin^2(a + \theta)}$$

when $\delta\theta$ is indefinitely diminished. The component of this at right angles to the vane is equal to

$$\frac{v_1 \sin a \cos(a + \theta) \cos\psi_1\, \delta\theta}{\sin^2(a + \theta)},$$

where $\psi_1 = \psi - (\theta - \theta')$ and the component of the last parallel to the direction of motion is

$$\frac{v_1 \sin a \cos(a + \theta) \cos\psi_1 \cos(\psi + a + \theta)\, \delta\theta}{\sin^2(a + \theta)}.$$

Now the weight of water per second passing over the strip between the cylinders, whose radii are R and $R + \delta R$ respectively, is equal to $\frac{W\, \delta R}{R_0 - R_1}$, and therefore the whole work done will be equal to

$$\frac{W v_1 \sin a}{g(R_0 - R_1)} \int_{R_1}^{R_0} \int_{\frac{\pi}{2} - a}^{\theta} \frac{\cos(a + \theta) \cos\psi_1 \cos(\psi + a + \theta)\, R\, d\theta\, dR}{\sin^2(a + \theta)}.$$

We have the following relations:

$$O'P^2 = pP^2 + O'p^2 - 2\,pP.\,O'p \sin(a + \theta);$$

also

$$pP^2 = \frac{\omega^2 R^2 r^2 \{\cos a - \cos(a + \theta)\}^2}{v_1^2 \sin^2 a}.$$

The equation to the relative path will therefore be

$$\frac{\rho^2}{r^2} = 1 + \frac{\omega^2 R^2}{v_1^2 \sin^2 a} \{\cos a - \cos(a + \theta)\}^2 - \frac{2\,\omega R}{v_1 \sin a}$$
$$\times \{\cos a - \cos(a + \theta)\} \sin(a + \theta); \qquad (1)$$

also, if θ' be the angular co-ordinate of the point P, we shall have

$$p\,P^2 = O'p^2 + O'P^2 - O'p \,.\, O'P\cos(\theta - \theta'),$$

or

$$\rho^2 + r^2 - 2\,\rho\, r\cos(\theta - \theta') - \frac{\omega^2 R^2 r^2 \{\cos a - \cos(a + \theta)\}^2}{v_1^2 \sin^2 a} = 0\;(2).$$

Theoretically equations (1) and (2) are sufficient to determine ρ and θ', and therefore $\tan\psi$ in terms of a, θ and R, and then after integration we might see if there was any maximum value of r in terms of a, ω, R. If we could ascertain the integral, we should not, however, obtain results of any value, since the oblique direction in which the jet leaves the vane, a point of by far the greatest importance, and which has already been fully discussed, is left out of consideration. We ought therefore to adhere to the value previously arrived at, viz.:

$$D_1 = D = D_0 = \cdot 25\, R_1.$$

The next point we have to discuss is the best value of the ratio $\omega\, R_1 \div v_1$, and in connection with this the probable theoretical vane efficiency. It is of course impossible to determine these without obtaining the integrals of the expressions for the work done, but we may arrive at an approximation to the best value by comparing the case of a turbine vane with that of a vane moving in a linear direction.

In outward-flow turbines, the tangent of the angle between the radius vector and the tangent to the relative path is equal to

$$\frac{R_0^2 - \rho^2}{\sqrt{4\rho^2 r^2 - (R_0^2 - \rho^2)^2}} - \frac{\omega\rho}{v_1 \sin a}.$$

Since the first term in this expression is equal to the tangent of the angle between the radius vector and the tangent to the absolute path, the absolute velocity at right angles to the radius vector is equal to

$$\frac{(R_0^2 - \rho^2)\, v_1 \sin a}{\sqrt{4\rho^2 r^2 - (R_0^2 - \rho^2)^2}};$$

therefore at the point where the tangent to the relative path coincides with the radius vector, the velocity of the vane in the direction of motion will be equal to the absolute velocity of the water in the same direction, and therefore no work will be done in passing over this element of the path.

We may therefore look upon the vane of a turbine as divided into two parts, and by varying the value of ω we may divide the whole head lost between the two halves of the vane in whatever proportion we like. The greater the velocity of the vane, the more nearly will the normal to the first part of the vane be parallel to the direction of motion, and therefore the greater the ratio effective power to power lost, but the less the whole power lost, and *vice versâ*. In the case of the second part of the vane, on the contrary, the greater the velocity, the greater will be the divergence of the normal to the vane from the direction of motion, and the less therefore the ratio of the effective power to the whole power lost, but the greater the whole power lost, and *vice versâ*. It is evident, then, that the effective power in direction of motion developed by the last part of the vane will be equal for two values of ω, the smaller value coinciding with the greater amount of head lost in passing over the first part of the vane. Therefore the greatest part of the effective work done will be due to the action of the first part of the vane, and if we determine the value of ω so as to make the work done by this a maximum, that value will probably make the whole work done by the vane a maximum.

Now we may look upon the motion during the interval between the jet striking the vane and reaching the point where $\tan \psi = 0$ as linear. If the angular velocity be determined so that the absolute velocity of the vane at the point where $\tan \psi = 0$ is equal to $\frac{v_1 \cos a}{2}$, the head lost in passing over the first half will be equal to the head lost when a jet strikes a flat vane moving at its best relative velocity, and this angular velocity will therefore probably be that of maximum efficiency for the turbine.

If we put

$$\frac{(R_0^2 - \rho^2)\, v_1 \sin a}{\sqrt{4\,\rho^2\, r^2 - (R_0^2 - \rho^2)^2}} = c\, v_1 \cos a,$$

we shall determine the value of ρ, which makes $\omega\,\rho = c\,.\,v_1 \cos a$, ρ being the radius at the point, where $\tan \psi = o$. Solving the equation,

$$\rho = \sqrt{\frac{r^2\, c^2 \cot^2 a}{1 + c^2 \cot^2 a} + R_0^2} - \frac{r\, c \cot a}{\sqrt{1 + c^2 \cot^2 a}}$$

when $\qquad a = 10°,\ \delta = 26°\ 57'\ 40'',\ r = 1\cdot 1372\, R_1$ and

$$\frac{\rho}{R_1} = \sqrt{\frac{41\cdot 593\, c^2}{1 + 32\cdot 163\, c^2} + 3\cdot 24} - \frac{6\cdot 449\, c}{\sqrt{1 + 32\cdot 163\, c^2}}.$$

If $\qquad c = \cdot 5$

$\rho = 1\cdot 02\, R_1,$

since the depth of the jet must be much less than the length of this part of the vane, it follows that in practice so small an angle as 10° cannot be adopted for the inclination of the guide blades, because the depth of the jet would have to be much less than one-fiftieth part of the radius. If

$$a = 20°, \ \delta = 22° \ 37' \ 24'', \ r = 1 \cdot 192 \, R_1$$

$$\frac{\rho}{R_1} = \sqrt{\frac{10 \cdot 72 \, c^2}{1 + 7 \cdot 545 \, c^2} + 3 \cdot 24} - \frac{3 \cdot 275 \, c}{\sqrt{1 + 7 \cdot 545 \, c^2}}$$

when $c = \cdot 5, \ \rho = 1 \cdot 08 \, R_1$.

This value is likewise too small for practical use. If the equation is examined, which gives the value of ρ in terms of c and r, it will be seen that either a decrease in the value of c, or an increase in the value of R_0, will increase the value of ρ. Take $c = \cdot 4$, then $\rho = 1 \cdot 1 \, R_1$, which is still too small. A less value cannot be given than this, which corresponds with the value $\omega \, R_1 = \cdot 34 \, v_1$, therefore the value of R_0 must be increased. The previous calculations are based on what has been proved must be the minimum value of the ratio $R_0 \div R_1$, viz., $1 \cdot 8$. Take $R_0 = 2 \, R_1$,

$$\delta = 26° \ 16' \ 30'', \ r = 1 \cdot 6 \, R_1$$

$$\frac{\rho}{R_1} = \sqrt{\frac{19 \cdot 32 \, c^2}{1 + 7 \cdot 545 \, c^2} + 4} - \frac{4 \cdot 396 \, c}{\sqrt{1 + 7 \cdot 545 \, c^2}}$$

when $c = \cdot 5$

$\rho = 1 \cdot 1 \, R_1$

when $c = \cdot 4$

$\rho = 1 \cdot 17 \, R_1$;

which is a suitable value. The corresponding value of $\omega \, R_1$ is $\cdot 32 \, v_1$. When $R_0 = 2 \, R_1$, the absolute velocity will be equal to $\cdot 4 \, v_1$, when $\rho = 1 \cdot 46 \, R_1$.

In the case of inward-flow turbines we may simplify the investigation by supposing the radial velocity constant and equal to $v_1 \sin a$, since up to the point where $\tan \psi = 0$, this could not differ appreciably from the truth. On this supposition

$$\tan \phi = \frac{-\rho}{\sqrt{4 r^2 - \rho^2}}$$

$$\tan \psi = \frac{\omega \rho}{v_1 \sin a} - \frac{\rho}{\sqrt{4 r^2 - \rho^2}};$$

therefore, when $\tan \psi = 0$, since $2r = R_1 \sec a$,

$$\omega \rho = \frac{\rho v_1 \sin a}{\sqrt{4r^2 - \rho^2}} = c v_1 \cos a$$

and

$$\rho = \frac{R_1 c}{\sqrt{c^2 \cos^2 a + \sin^2 a}}$$

as in outward-flow turbines we may show that we cannot adopt a less value for a than 20°. With this value we get

when $c = \cdot 5$

$$\rho = \cdot 86 R_1, \omega R_1 = 53 v_1,$$

when $c = \cdot 4$

$$\rho = \cdot 8 R_1, \omega R_1 = \cdot 47 v_1.$$

In the case of parallel-flow turbines we cannot obtain a solution of the problem, but the best value of the ratio $\omega R \div v_1 \cos a$ cannot differ much from $\cdot 5$. We should probably find also that we could not adopt in practice a less value for a than 20°.

We are now in a position to discuss the value of the co-efficient of efficiency, in arriving at which we shall have to examine—

(1.) The magnitude of the vane efficiency estimated on the supposition of the exact truth of the assumptions on which the calculations are based.

(2.) The divergence of the actual from this calculated theoretical efficiency, owing to the divergence of the assumptions from exact truth.

(3.) The ratio of the efficiency of the wheel to that of the vane.

In estimating the magnitude of the vane efficiency, we must consider the two parts separately. As regards the first part, since the head lost in passing over any element may be divided into two components, one parallel to the direction of motion, and the other radial, which last can have no effect on the motion, but simply causes a radial strain towards the centre in outward, and from the centre in inward-flow turbines, it is evident that the efficiency must be less than that of a flat vane moving in a linear direction inclined at angle a to that of the jet, or the theoretical efficiency must be less than $\frac{\cos^2 a}{2}$. Since the value of the ratio $\omega R_1 \div v_1 \cos a$ differs more from its best value in the case of outward-flow turbines than in that of inward, this part of the efficiency will probably be less in the former case than in the latter.

As regards the second half of the vane, the greater the height due to the velocity with which the jet leaves the first half, the greater will be the head lost in passing over the second half.

In the case of outward-flow turbines we cannot have $\omega R_1 > \cdot 32\ v_1$ unless a be $> 20°$, and the height due to the absolute velocity on leaving the first half will be about

$$\frac{(\cdot 16 \cos^2 a + \sin^2 a)\, v_1^2}{2g} = \frac{\cdot 25\, v_1^2}{2g},$$

and the absolute velocity itself will be equal to $\cdot 5\ v_1$.

In the case of inward-flow turbines, on the contrary, we may have the initial vane velocity equal to $\cdot 53\, v_1$, and the height due to the velocity on leaving the first half of the vane will be equal to

$$\frac{(\cdot 25 \cos^2 a + \sin^2 a)\, v_1^2}{2g} = \frac{\cdot 34\, v_1^2}{2g},$$

and the absolute velocity itself will be equal to $\cdot 58\ v_1$.

Thus the head lost in passing over the second part of the vane of an inward-flow turbine must be about 10 per cent. more than that lost in the case of an outward-flow turbine, and therefore the efficiency in the former case somewhat greater than in the latter, if the ratio effective power ÷ power lost be identical in the two cases.

In the case of an outward-flow turbine the maximum value of the head lost in passing over the second half of the vane would therefore be equal to $\frac{\cdot 14\, v_1^2}{2g}$, and in that of an inward-flow turbine to $\frac{\cdot 23\, v_1^2}{2g}$. Now if this head were lost by impact against a flat vane moving normally the effective power developed would be equal to

$$\frac{\cdot 66 \times \cdot 14\, v_1^2}{2g} = \frac{\cdot 09\, v_1^2}{2g}$$

in the first case, and in the second to

$$\frac{\cdot 66 \times \cdot 23\, v_1^2}{2g} = \frac{\cdot 15\, v_1^2}{2g}.$$

Since, however, a great part is lost in producing radial pressure the effective power developed by the vanes of turbines must be much less. The efficiency of outward-flow turbines must therefore be less than ·56, and of inward-flow less than ·62.

By sufficiently increasing the number of the vanes and guide blades we may make the divergence from exactness of the assumptions on which the theoretical investigations are based practically unappreciable, whilst in the case of the flat vanes of a vertical wheel the jet strikes the vanes at various angles and with less than the assigned initial velocity owing to the increment in the area of the jet at some distance before it strikes the vane. In addition, since the jet strikes and glances off the vane at the same side, the recoil of the water which first stikes the vane will agitate and diminish the velocity of that which last reaches it. On the whole, therefore, we may conclude that the ratio of the actual to the theoretical vane efficiency in the case of a turbine vane is much greater than in that of the vane of a vertical wheel. If we take the actual vane efficiency, therefore, of a turbine as equal to the theoretical vane efficiency of a vertical wheel working clear of tail water, we shall have the efficiency of the former to that of the latter = 5 : 4 nearly.

The ratio of the efficiency of the wheel to that of the vanes depends upon the amount of the resistance due to friction of bearings and of the air or water in which the wheel revolves. Some experiments made to determine the difference between the efficiency of turbines working in air and working in water, made by Monsieur Girard, show that the efficiency in the latter case is only about 85 per cent. of the efficiency in the former. The friction due simply to bearings is very trifling, so that if the turbines work in air, we may consider the wheel efficiency practically equal to the vane efficiency. The actual efficiency will probably therefore lie between ·40 and ·55.

Although it does not lie within the scope of the present Paper to give any description of the different sorts of turbines which have been invented, the Author cannot avoid discussing one particular turbine invented by Professor Thompson, and called by him the "vortex turbine," because the principles of construction and the nature of the action of the water against the vanes, as described by that gentleman, are diametrically opposed to those laid down and described in this Paper.

The following description of the principles of construction of this turbine is abridged from a Paper read by Professor Thompson before the British Association in 1852:—

"The velocity of the circumference is made the same as that of the entering water, and thus there is no impact between the water and the wheel. This velocity must be equal to the velocity which a heavy body would attain in falling through a vertical space equal to half the height of the fall. Thus one half only of the fall is employed in producing velocity in the water, and therefore the other half still remains acting on the water within the wheel chamber at the circumference of the wheel in the condition of fluid pressure. Now with the velocity already assigned to the wheel it is found that this fluid pressure is exactly that which is requisite to overcome the centrifugal force of the water on the wheel and to bring the water to a state of rest at its exit, the mechanical work due to both halves of the fall being transferred to the wheel during the combined action of the moving water and the moving wheel. In the foregoing statements the effects of fluid friction and of some other modifying influences are for simplicity's sake left out of consideration. Thus by the balancing of the contrary fluid pressures, due to half the head of water and the centrifugal force of the water in the wheel, combined with the pressure due to the ejection of water backwards from the inner ends of the vanes of the wheel when they are curved, only half the work due to the fall is spent in communicating *vis viva* to the water, to be afterwards taken from it during its passage through the wheel, the remainder of the work being communicated through the fluid pressure to the wheel without any intermediate generation of *vis viva*. Thus the velocity of the water where it moves fastest in the machine, is kept comparatively low, not exceeding that due to half the height of the fall, while in other turbines the water usually requires to act at much higher velocities. In many of them it attains at two successive points the velocity due to the whole fall."

The above is a word-for-word abridgment, that is to say, it is an extract with intermediate parts not directly bearing on the main principles of action left out, not a summary of Professor Thompson's Paper. In it there are three salient principles enunciated which are diametrically opposed to those already laid down in this Paper. These are:

1st. That a velocity of rotation is communicated by the turbine to the water.

2nd. That statical fluid pressure can develop effective power in a turbine.

3rd. That in any turbine the absolute velocity can at two successive intervals be equal to that due to the whole fall.

To controvert the first statement it seems almost sufficient to point out that the water produces rotary motion in the turbine, not the turbine in the water. It is clear, however, that whatever change of absolute velocity the vanes of the turbine may produce in the water, it is only their relative motion which we have to discuss. This will be the same if velocities equal and opposite to that of each point of the vane be impressed on both the vane and the water so as to bring the vane to rest. The water now clearly can have no motion of rotation, and therefore no centrifugal force can be impressed upon it. Since the relative motion is the same in both cases there will be no motion of rotation and no centrifugal force communicated by the vanes to the water when the turbine is in motion. If some extraneous force, other than that due to the impulse of the water, were applied causing the turbine to rotate with any given angular velocity, and the water entered the drum radially, the breadth of the crown, that is the difference between the external and internal radii, which would contain a sufficient weight of water, moving with the assigned angular velocity, to reduce the radial velocity of the entering water by an amount equal to that due to half the whole head, is a matter readily ascertained. The external power so employed must be greater than the effective power of half the fall, since the effective power developed at the periphery of the turbine is equal to the gross power of half the fall. We see, then, that the power of half the fall cannot be applied by any machine of which the efficiency is not perfect to counteract the power of the remaining half in the manner indicated.

It follows, then, *a fortiori*, that if the power due to the first half of the fall could produce such a velocity of rotation on the mass of water in the turbine as to counterbalance the power due to the remaining half, no external effective power would be developed by a turbine under those conditions unless it be developed by the statical pressure of the fluid against the vanes. Now since such a pressure would act equally against the front of one vane and the back of the succeeding one, it follows that this pressure could have no effect in producing motion. As to the statement that in some turbines the absolute velocity at two successive points can be equal to that due to the whole fall, it is merely necessary to remark that as no head can have been lost in the interval, no effective power can have been developed. Such a turbine may exist, but the part of the machine intervening between the two points is manifestly useless. If no friction existed, or if its amount could be ascertained, there would be no theoretical difficulty in designing an outward-flow turbine in which the relative velocity of the water at a given point should be equal to its initial absolute velocity.

Since, theoretically, in a "vortex turbine" the initial absolute velocity is due only to half the whole head, the area of discharge should be to that of turbines in which the initial velocity is that due to the whole head as 7 : 5. If, therefore, no such centrifugal force exists as that described by Professor Thompson, the actual discharge will be to the estimated discharge as 7 : 5, and the actual efficiency to the estimated efficiency as 5 : 7.

Very few turbine-makers hesitate to guarantee 75 per cent. efficiency. Some even do not stop short of 80 per cent. The conclusions at which the Author has arrived, as regards the value of the efficiency, differ so widely from this, that no possible divergence of the conditions stated in the assumptions, on which the calculations are based, from what actually takes place, can save us from this dilemma. Either those assumptions are utterly wrong in principle, or the results of the experiments are utterly unreliable, because if the assumptions be true in principle any deviation from them will diminish, not increase the efficiency.

The Author has not met with any record of experiments made on small models, where the exact amount of water can be ascertained with accuracy, but only of those made with wheels constructed to utilise large volumes of water. In these cases the correctness of the results depends entirely on the accuracy of the gauging, which virtually means that we may, within by no means narrow limits, make the efficiency what we like.

In the Paper of Professor Thompson's already referred to there is a tabular statement of thirteen experiments made by him with a wheel at Ballysillan. The gauging was made on a rectangular weir 3 feet long, the co-efficients used being those of Poncelet and Lesbros. The Paper does not give a description of the nature of the weir, nor does it state the area of the openings between the guide blades of the turbines. The heads in the first column are the depths of the water measured over the weir, not the depth of the crest below the still water, so that in using Poncelet and Lesbros' co-efficients, which all refer to the head measured from still water, we must make an allowance for the height due to the surface velocity at the crest. Since the less the width of the crest the less will be the height due to the velocity of approach, we shall arrive at the smallest calculated quantity by assuming that the gauging was made by means of a plank weir.

The formula for ascertaining the theoretical discharge over a weir, in cubic feet per second, where the velocity of approach is neglected, is

$$D = \frac{2}{3} l h_w \sqrt{2 g h_w},$$

where D is the discharge, and l, h_w the length of the weir and depth over the weir in feet. If we compare the theoretical quantities obtained from the formula with those given by Professor Thompson, we find that he has used the co-efficient ·612, which is that given by Poncelet and Lesbros for heads measured from still water for sunk orifices, in which the ratio of the length to the depth is the same as that of the notch we have to gauge. If, then, h represents the height of the surface of still water above the crest, the gauging, according to Poncelet and Lesbros, would be

$$D = \cdot 613 \times \frac{2}{3} l \sqrt{2 g} \left\{ h^{\frac{3}{2}} - (h - h_w)^{\frac{3}{2}} \right\}.$$

For a similar ratio of length to depth of notch Brindley and Smeaton give the co-efficients for heads measured from still water ·657, Du Buat ·627, and Messrs. Simpson and Blackwell ·756, the ratio of the length to the depth of the notches in the last case being about 10 to 1.

In the above list of co-efficients the small ones are those determined from experiments with small depths of notches from 1″ to 2″, in which the loss from friction and contraction must necessarily be greater in proportion to the whole discharge than when the notches are deep. In Simpson and Blackwell's experiments the depth over the crest was about equal to that in the gauging made by Professor Thompson.

The next point we have to consider is the height due to velocity of approach. Du Buat's experiments led him to conclude that the head over the weir was only equal to one-half the head measured from still water. For heads from still water of about 7 inches Messrs. Simpson and Blackwell found that $h = 1 \cdot 3\ h_w$. The minimum value appears to be $h = 1 \cdot 2\ h_w$. This value has been used in the following table to correct the gauging made by Professor Thompson, and the value $h = 1 \cdot 25\ h_w$ to determine the gauging according to Brindley and Smeaton's co-efficients.

In some experiments made by Mr. Ballard at Worcester, ·75 was determined as the proper co-efficient for heads measured on the crest, which agrees with the corrected co-efficient of Poncelet and Lesbros. Du Buat's co-efficient is the same as that of Messrs. Simpson and Blackwell, ·94.

In the following table the heads over the weir varied from ·617 to ·718 feet, and the theoretical discharge, neglecting velocity of approach, from 524 to 579 cubic feet per minute.

—	Co-efficient of discharge over Weir.	Co-efficients of Efficiency.													Average co-efficient.
Number of Experiment.	—	1	2	3	4	5	6	7	8	9	10	11	12	13	—
Prof. Thompson's co-efficients .	·613	·75	·74	·78	·76	·77	·67	·76	·75	·63	·68	·68	·75	·74	·73
Poncelet and Lesbros .	·75	·60	·59	·63	·61	·62	·54	·61	·60	·50	·55	·55	·60	·59	·59
Brindley and Smeaton .	·83	·56	·55	·58	·56	·57	·50	·56	·55	·47	·51	·51	·56	·55	·54
Simpson and Blackwell	·94	·49	·48	·51	·49	·50	·44	·49	·49	·41	·44	·44	·49	·48	·47

The co-efficient of efficiency, therefore, may have been as low as ·41, and cannot have been higher than ·63. The average of three sets of gaugings, in which the velocity of approach has been allowed for, is ·53.

Since the gauging is a matter of so much uncertainty, it is evident that, even if we knew the exact quantity of water available, we could not design the guide-blade orifices with sufficient exactness. If too large, head would be lost, if too small, the quantity of water passing through the turbine would be less than the available quantity. In addition, therefore, to the necessity of being able to vary the quantity of water to suit the varying amount of work to be done, we must have the means of varying the area of the guide-blade orifices, so as to insure the passing of the whole of the water without losing head. In practice this is done in two ways—either by using movable guide blades, or by having sluices in front of the guide-blade openings. It is quite evident that there must be a loss of efficiency in the case of movable guide blades whenever the inclination of these to the direction of motion differs from that in accordance with which the curve of the relative path has been designed. Therefore movable guide blades ought not to be used, because the efficiency will be least at a time when it ought to have its maximum value, viz., when the quantity of water is least.

When sluices are used there may be either a separate sluice for each orifice, a separate sluice for every two or three or more, or one sluice for the whole number. When there is only one sluice it is evident that as the sluice closes, the area of flow on the receiving side of the guide-blade chamber diminishes, whilst that on the discharging side remains constant. The velocity of flow, therefore, on the receiving side will increase, and that on the discharging side decrease, as the sluice closes, until they become equal. After this point the velocity on the receiving side will exceed the velocity on the discharging side. In addition to this, the motion of the water, when it strikes the vanes, will not be steady, and the issuing water will form eddies between the vanes with the external water, which will have a tendency to force its way into the turbine. Therefore the efficiency, with such a sluice, will diminish very rapidly as the sluice closes. Fourneyron, in some experiments made at Inval, found that the efficiency, when such a sluice was lifted one quarter the full height, was only equal to five-sevenths of the efficiency when the sluice was open. To remedy this he inserted horizontal diaphragms between the guide blade and the vanes. It is evident that these could only be of much efficiency when the bottom of the sluice was exactly opposite to one of them, and the loss by friction must have been very much increased when the sluice was fully opened.

The greater the number of sluices the fewer will be the number of guide-blade orifices affected by the diminished velocity of discharge, and consequently the less the diminution of efficiency owing to the diminished quantity of water. When some of the guide-blade orifices are closed there will probably be a slight increase in the frictional resistance of the external water, because the water opposite the closed guide blades will not, like the issuing water, be moving with the relative velocity of the water and the vane.

To secure a maximum of efficiency there ought to be as many guide blades as vanes, and the interval between consecutive guide blades should be as small as is practically possible.

APPENDIX. I.

Summary of Smeaton's Experiments.

The following description of the mode of performing his experiments, and the tabular statement of the results, are taken from a Paper read before the Royal Society in 1759 on the natural powers of wind and water. The wheel, which was 24 inches diameter, had flat radial vanes. It revolved in a race into which the vanes exactly fitted, leaving just sufficient play between the bottom and the sides to prevent any chance of the wheel being impeded by contact with these and the vanes. At the upper end of the race was a cistern provided with a sluice capable of being lowered or raised at the will of the operator. The water after leaving the wheel fell into a receiver, whence it was pumped up into the cistern. The apparatus was so arranged that the operator could keep the water in the cistern during the experiment at the same level, so that the force exerted by the issuing water remained constant during each experiment. The quantity of water issuing per second was exactly computed by the number of strokes of the pump.

The water struck the vanes at the lowest part of the wheel on a level with the sluice, and therefore the current must previously have moved along the race a space greater than the radius of the wheel, so that its velocity at the instant of impact could not be even approximately calculated by theory. Its exact value was ascertained experimentally in the following manner:—Around one end of the axis a hollow drum was attached capable of engaging and disengaging itself by means of a clutch. Round this drum was wound a string connected with a system of pulleys, and a scale for weights. The string could be wound round any side of the drum, so as to act either with or against the stream. As a first approximation, the number of revolutions of the wheel were counted with no weight in the scale. Since it would be retarded by the weight of the scale and the frictional and other resistances, the actual velocity would be somewhat greater than the speed of the circumference under these conditions. The water was now shut off, and the string wound round the drum so as to turn the wheel in the same direction as the current. Weights were then put into the scale so as to make the wheel revolve somewhat faster than it had done under the action of the water. Now, it is evident that when the water is again admitted, the motion of the wheel will be accelerated, if the speed of its periphery be less than that of the current, and retarded if greater, and if equal, equal.

By this means, then, the actual velocity of the water at the instant of impact was ascertained.

Smeaton's object being to ascertain the efficiency of the vane apart from that of the wheel, it was necessary to clear the results obtained from the effects of friction and other resistances, the principal being the action of the air against the vanes. He achieved this as follows:—After having found out the speed which gave the best results, he shut the water off, and applied such a weight to the scale as made the wheel turn the same number of revolutions. The weight of the scale, then, and this added weight, are due to the frictional and other resistances, and therefore to ascertain the full value of the effective power of the impulse on the vane these must be added to the weights put into the scale, and the weight of the scale itself. For the sake of illustrating his Table, Smeaton has given the following example of the working out of one of his experiments.

SPECIMEN OF A SET OF EXPERIMENTS.

Sluices drawn to first hole.	
Water above floor of sluice	30 inches
Strokes of pump per minute	39½
Head raised by twelve strokes	21 inches
Area of head	105·8 sq. inches
Wheel raised empty scale, and made turns in a minute	80
With counter-weight of 1 lb. 8 ozs. it made .	85
With ditto tried with water	86

No. of Experiment.	1	2	3	4	5	6	7	8	9
Weight in lbs. & ozs.	4 0	5 0	6 0	7 0	8 0	9 0	10 0	11 0	12 0
Turns in a minute .	45	42	36¼	33¾	30	26½	22	16½	0
Product	180	210	217½	236¼	240	238½	220	181½	0

Counter-weight in the scale for 30 turns .	2 ozs.
Weight of empty scale and pulley . . .	10 ozs.
Circumference of cylinder	9 inches
Circumference of water-wheel	75 inches

From the above results of experiment we find by calculation power of water.

Head due to velocity of impact	15 inches	
Weight of water expended in a minute . .	264·7 lbs.	
Power of water in pounds and inches . .	3,970	
Effective work done.		
	lbs	ozs.
Weight in scale at maximum	8	0
Weight of scale and pulley	0	10
Counter-weight of scale and pulley (value of wheel resistance)	0	12
Sum of resistances	9	6
Height raised	135 inches.	
Effective power exerted in pounds and inches	1,266	

Whence $\frac{\text{Effective power}}{\text{Whole power}}$ . . . . $= \cdot 318$

$\frac{\text{Effective power}}{\text{Power lost}}$ . . . . $= \cdot 362$

$\frac{\text{Best velocity of vane}}{\text{Velocity of current}}$ . . . . $= \cdot 35$

In estimating the ratio of the best load to the maximum load, Smeaton entirely neglects the frictional and other resistances in the case of the maximum load for the following reasons, assigned in a foot-note:—"The resistance of the air in this case ceases, and the friction is not added, as 12 lbs. in the scale was sufficient to stop the wheel after it had been in full motion, and therefore somewhat more than a counterbalance for the water."

The resistance of the air ought, of course, to be omitted, but surely not the resistance of friction, since 12 lbs. is not the weight which suddenly stops the wheel, but is the weight arrived at by successive increments of 1 lb., which keeps the wheel at a state of rest, and must therefore be increased by the value of the load due to friction. The omission of this load, which in the case of small velocities is nearly equal to that in the scale, makes the maximum load nearly equal to the whole load, and causes him to remark:—"It is somewhat remarkable that, though the velocity of the wheel in relation to the water turns out to be greater than $\frac{1}{3}$rd the velocity of the water, yet the impulse of the water in the case of a maximum is more than double what is assigned by theory; that is, instead of being $\frac{4}{9}$ths of the column, it is nearly equal to the whole column."

The Author has already demonstrated in Part I. that the correct theoretical value of the maximum load is equal to two-thirds of the pressure on the vane when stationary. Owing to the effect of friction being left out of consideration, the experimental values assigned by Smeaton are much greater than this. Fortunately, the example given of the way in which the Table is formed affords the means of ascertaining the value of this frictional resistance with sufficient exactness. Since friction is approximately independent of the velocity, although it will be somewhat greater for heavy loads on the scale than light loads, we may consider it practically constant, and that the variation in the total resistance is wholly due to the resistance of the atmosphere, which will vary as the square of the angular velocity of the wheel. The value of the friction thus determined will be less than the correct value, and therefore the value of the ratio best load ÷ maximum load still too great.

If x represent the load due to friction, and y the load due to air resistance for 30 revolutions, the resistance due to 85 revolutions will be equal to $\left(\frac{85}{30}\right)^2 y = 8\,y$ nearly, and we shall have

$$\begin{aligned} x + y &= 12 \text{ ozs.} \\ x + 8\,y &= 34 \text{ ozs.} \\ y &= 3 \text{ ozs. nearly} \\ x &= 9 \text{ ozs.} \end{aligned}$$

Now, this frictional resistance is due to the weight of the wheel and the weights of the scale and pulley and the added weights. These last are probably very much less than the weight of the wheel itself, so that there could not be a material increase in the value of x due to the weights in the scale. We may, therefore, take 10 ozs. as a fair approximate value. Column 13a, added by the Author, gives the corrected values of the ratio load at best speed ÷ by load at equilibrium. When the added loads are large, the corrected ratios do not differ much from those given by Smeaton, but differ materially when the loads are small, for the reason already assigned, that the load due to friction in the latter case is much more nearly equal to the whole applied load than in the former. Notwithstanding this, Column 13a still presents the curious anomaly of the ratio in question being greatest when the experimental value of the ratio best velocity of vane ÷ velocity of current is greatest. These last values occur when the virtual head and the loads in the scale are least, and are plainly owing to the assumed value, viz., 10 ozs. of the load due to friction being less than its real value. Column 11a, giving the ratio effective power ÷ power lost, has also been added by the Author.

In the beginning of his Essays, Smeaton states that he found the results of actual practice agree very approximately with those of his experiments. In the latter the wheel efficiency of the model was equal to about 90 per cent. of the experimental vane efficiency, or the efficiency of undershot wheels working drowned is equal to about ·27.

No. of Experiment.	Height of Water in Cistern in inches.	Turns of Wheel unloaded.	Virtual head in inches deducted from 3.	Turns of Wheel at best speed.	Load at the equilibrium.		Load at the best speed.		Water expended p… minute in lbs.	Power.	Effect.	Effect / Power.	Effect / Power lost.	Best velocity of V… / Velocity of Wat…	…speed. / Load at equilibrium. Neglecting friction of Wheel.	…speed. / Load at equilibrium. Taking account of Wheel friction.	Left of Sluice.
1	2	3	4	5	6		7		8	9	10	11	11*a*	12	13	13*a*	14
					lbs.	oz.	lbs.	oz.									
1	33	88	15·85	30	13	10	10	9	275	4358	1411	·324	·48	·34	·775	·73	At 1st hole (1–10)
2	30	86	15	30	12	10	9	6	264·7	3970	1266	·32	·46	·35	·74	·71	
3	27	82	13·7	28	11	2	8	6	243	3329	1044	·315	·46	·34	·75	·71	
4	24	78	12·3	27·7	9	10	7	5	235	2890	901·4	·312	·45	·355	·753	·71	
5	21	75	11·4	25·9	8	10	6	5	214	2439	735·7	·302	·43	·345	·732	·68	
6	18	70	9·95	23·5	6	10	5	5	199	1970	561·8	·285	·40	·336	·802	·73	
7	15	65	8·54	23·4	5	2	4	4	178·5	1524	442·5	·29	·40	·36	·83	·73	
8	12	60	7·29	22	3	10	3	5	161	1173	328	·28	·38	·377	·91	·77	
9	9	52	5·47	19	2	12	2	8	134	733	213·7	·29	·40	·365	·91	·74	
10	6	42	3·55	16	1	12	1	10	114	404·7	117	·282	·40	·38	·93	·70	
11	24	84	14·2	30·75	13	10	10	14	342	4890	1505	·307	·44	·366	·79	·76	At 2nd hole. (11–17)
12	21	81	13·5	29	11	10	9	6	297	4009	1223	·301	·44	·362	·805	·76	
13	18	72	10·5	26	9	10	8	7	285	2993	975	·325	·40	·36	·875	·81	
14	15	69	9·6	25	7	10	6	14	277	2659	774	·292	·41	·362	·9	·83	
15	12	63	8	25	5	10	4	14	234	1872	549	·294	·41	·397	·87	·78	
16	9	56	6·37	23	4	0	3	13	201	1280	390	·305	·44	·41	·95	·85	
17	6	46	4·25	21	2	8	2	4	167·5	712	212	·298	·42	·455	·9	·73	
18	15	72	10·5	29	11	10	9	6	357	3748	1210	·323	·47	·402	·805	·76	At 3rd hole. (18–21)
19	12	66	8·75	26·75	8	10	7	6	330	2887	878	·305	·43	·405	·86	·80	
20	9	58	6·8	24·5	5	8	5	0	255	1734	541	·301	·46	·422	·91	·81	
21	6	48	4·7	23·5	3	2	8	0	228	1064	317	·299	·41	·49	·96	·80	
22	12	68	9·3	27	9	2	8	6	359	3338	1006	·302	·43	·397	·917	·86	At 4th hole. (22–24)
23	9	58	6·8	26·25	6	2	5	13	332	2257	686	·304	·43	·452	·95	·87	
24	6	48	4·7	24·5	3	12	3	8	262	1231	385	·313	·45	·51	·935	·80	
25	9	60	7·29	27·3	6	12	6	6	355	2588	783	·303	·43	·455	·945	·86	At 5th hole. (25–26)
26	6	50	5·03	24·6	4	6	4	1	307	1544	450	·292	·41	·49	·93	·81	
27	6	50	5·03	26	4	15	4	9	360	1811	534	·295	·42	·52	·925	·80	At 6th hole. (27–28)
28	Theoretical values.											·385	·58	·58	..	·67	

APPENDIX II.

SECTION I.—FURTHER REMARKS ON THE MAGNITUDE OF THE RELATIVE VELOCITY BEFORE AND AFTER IMPACT, AND ON THE STATICAL MEASURE OF THE IMPULSIVE PRESSURE EXERTED BY A JET AGAINST A VANE.

The preceding investigations in this work are based on the assumption that the jet of liquid consists of an infinite number of indefinitely small inelastic molecules. In accordance with this theory no change of velocity can take place except normally to the plane of impact, and that change will be equal to the relative velocity of the jet and vane estimated in that direction.

The theoretical values of the work done by a jet striking a vane obtained by calculations based on this theory are not strictly true, because it is assumed that each molecule actually strikes the vane at the same angle as the axis of the jet cuts the normal to the vane. In reality the vane spreads out into a conical shape, so that the different threads, of which we may suppose it is comprised, are inclined at various angles to the plane of impact, and only a part of the jet actually strikes the vane, the impulsive pressure impressed on the vane by the remainder being conveyed through the agency of the intervening fluid. In accordance with this theory, in order to insure a maximum of efficiency, the sluice orifice ought to be as small as can practically be adopted.

The other theory states that the relative velocity after impact is equal to the relative velocity before impact. If then the jet strike the vane without shock, so that no velocity is lost in whirls and eddies, in accordance with this theory, if tangential friction be left out of consideration, the power lost by the water will be exactly equal to the power imparted to the vane. It is, in fact, an extension of the maxim of the chemists, that matter itself is indestructible, to the temporary qualities or properties possessed by matter.

An analogous case would be that of two inelastic masses m, m', impinging against one another. Suppose, for the sake of simplicity, that the mass m' is at rest before impact. If the mass m be moving with a velocity v before impact, its capacity for work will be equal to $\frac{m\,v^2}{2}$. After impact the two masses move on in contact with a common velocity v which is equal to $\frac{m\,v}{m+m'}$, so that the sum of the capacity for work finally possessed by the mass m plus that possessed by the mass m' will be equal to $\frac{m^2\,v^2}{2\,(m+m')}$, which is always less than the original capacity possessed by m, and may be indefinitely small. If the balls are elastic and of the same substance, whose modulus of elasticity is e, we shall have the velocity of the mass m after impact equal to $\frac{(m - e\,m')}{m+m'}\,v$, and that of m' equal to $\frac{(1+e)\,m\,v}{m+m'}$, so that the final capacity for work is equal to $\frac{v^2\,m\,(m+e^2\,m')}{2\,(m+m')}$, which in one case only is equal to $\frac{m\,v^2}{2}$, viz., when the elasticity is perfect.

It may be that the principle, properly interpreted, is correct, and that the power lost by the first ball is partly expended in communicating momentum to the second and partly in producing heat, which, in the case of perfectly elastic balls, is at once given back again in the shape of an increase of the momentum due to direct impact, and in the case of inelastic balls remains stored up, and simply increases the temperature of the masses m and m'. Any discussion as to the relative temperatures of the water and the vane after and before impact is, however, irrelevant. The Author will have gained his point if he can prove, either by argument or experimental results, that conclusions as to the efficiency of water-power machines, based on the assumption—to use Viry's own words—"*rien se perd dans la nature*," must be erroneous, when that maxim is interpreted to mean that the effective power communicated to the machine is equal to the power lost by the water.

In the memoir the Author has proved, that the relative velocity after impact cannot be the same as the relative velocity before impact, if it be true that no reaction can take place between a jet and a smooth vane, except in a direction normal to the plane of contact. Leaving theoretical reasoning aside, the truth or falsity of the opposite assumptions may readily be ascertained by very simple experiments.

If the flat smooth plate of metal attached to the end of the hose of the common street-watering machines be held horizontal, and the nozzle of the jet likewise at first held horizontal and subsequently lowered so as to strike the plate at different angles, it will be found that the range of the jet rapidly decreases as the angle of inclination of the jet to the vane increases.

Again, if a jet whose initial velocity is due to a head of 4 or 5 feet be directed horizontally against a smooth vertical wall, it will be found that the water rises vertically on the wall, not 4 or 5 feet, but only as many inches. Mere friction in this case could not have destroyed from 80 per cent. to 90 per cent. of the power.

In fact, if the assumption is even approximately true, it would not be possible to fill from a tap any smooth vessel with diverging sides.

Monsieur Viry, in the treatise to which reference has already been made, has given an investigation into the value of the statical measure of the impulse against vanes at rest, of which the following is a summary. It is based on the assumption that the theory of Bernouilli, demonstrated to be true under certain conditions only, when applied to the case of water flowing with steady motion along a tube or channel, is applicable to the case of a jet striking a vane, and that the relative velocity after impact is equal to the relative velocity before impact.

In Figs. 1, 2, and 3, R X is normal to the vane G P vertical, β is the angle between the normal and the vertical, α between the normal and the direction of the jet, and α' the angle of inclination of the verge boards in Figs. 2 and 3 to the normal.

If we consider the change of motion of the volume of water contained between A B, C D, and E F as it passes into the position A′ B′, C′ D′, and E′ F′, it is evident that the dynamical state of the part A′ B′, C D, E F common to the two positions of the given mass of fluid is unaffected by the change, and we have then only to consider the change of motion of the volumes A B A′ B′, C D C′ D′, and E F, E′ F′.

In accordance with Bernouilli's theorem, no account is to be taken of the internal molecular agitations, but only of the initial and final states of the liquid.

Case I.—Plane Vane, Fig. 1.

Since the motion of the water finally in the parts C D, C′ D′, and E F, E′ F′ is wholly tangential, there is no change of momentum normal to the plane.

If v be the velocity and p the weight of the water in the part A B A′ B′, the initial normal momentum will be equal to $\frac{p}{g} v \cos \alpha$. Since this is directed from

X towards R it will have a negative sign, and the variation of the momentum projected on the plane will be

$$o - \left(\frac{-p\ v \cos \alpha}{g}\right) = \frac{p\ v \cos \alpha}{g}.$$

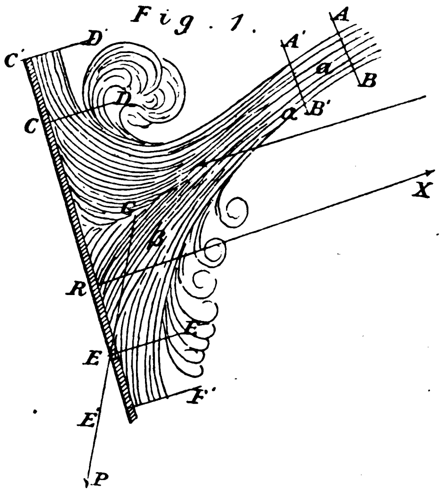
We have now to consider the value of the normal impulses which have produced the above change of motion. These are due to the normal component of the weight P of the volume A′ B′ C D E F and the reaction R of the plane. Since the component of the weight acts from X towards R, it must be affected with a negative sign, and the sum of these impulses during an interval θ will be

$$- P\ \theta \cos \beta + R\ \theta,$$

and we obtain, by equating these with the expression for the change of motion,

$$R\ \theta = P\ \theta \cos \beta + \frac{p\ v \cos \alpha}{g}.$$

Fig. 1.



If ω be the area of the jet at A B, and π the weight of an unit of volume, we get

$$p = \pi\ \omega\ v\ \theta$$

$$R = P \cos \beta + \frac{\pi\ \omega\ v^2 \cos \alpha}{g}.$$

Cases II. and III.—Vanes with verge boards, Figs. 2 and 3.

The investigation in these cases is precisely similar, but the final velocity is $v \cos \alpha'$ in direction of R X in the former, and from X to R in the latter, so that we get

$$\text{Case II.} \quad R = P \cos \beta + \pi\ \omega\ v^2 \frac{(\cos \alpha + \cos \alpha')}{g}$$

$$\text{Case III.} \quad R = P \cos \beta + \pi\ \omega\ v^2 \frac{(\cos \alpha - \cos \alpha')}{g}.$$

In the above expressions for the resistance overcome by a flat vane in an unit of time there are two elements—the statical weight of the superincumbent water, and the statical measure of the momentum exerted by the water during the same interval: since the former is constant and independent of the time, it

Fig. 2.

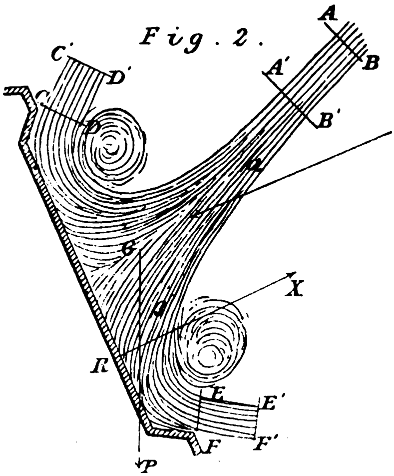
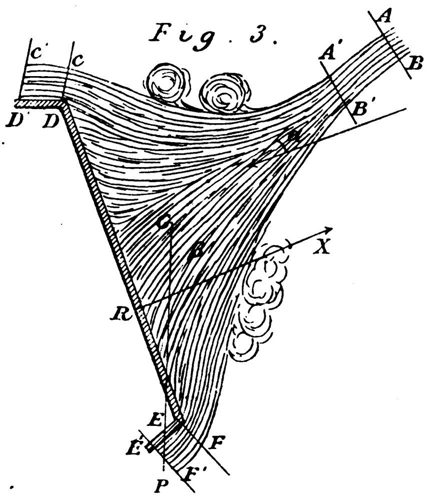


Fig. 3.



clearly cannot be a member of an equation of relation between the resistance overcome in an unit of time and the momentum of the water expended in that time. Apart from this consideration the equation does not contain the correct value of the superincumbent weight. Since the demonstration above given holds perfectly true at whatever points we take the volumes A B A′ B′, C D C′ D′, and E F E′ F′ so long as at the points selected the motion is steady and unaffected by the impulse, and in the last parallel to the plane of the vane, it follows that, according to Viry's theory, we may give to R any value we like. It is plain, however, that the dead weight of the water acting normally against the vane must be due to the volume of water in contact with the vane, of which the normal velocity is wholly destroyed, and will therefore vary with the area of the vane, but will be perfectly independent of the volume A′ B′, C D, E F.

The value of the direct impulsive resistance overcome is the same as that given in this memoir, because the value of the final relative velocity does not affect the problem in the case of a simple flat vane.

In Case II., although the reaction can only take place in one direction, viz., normal to the plane of the verge boards, the component of the resistance overcome by the latter at right angles to the original plane of impact is stated to be equal to $p\ v \cos \alpha'$. Its correct value is clearly equal to $p\ v \cos \alpha' \sin \alpha$, since the water moving along the initial plane of impact parallel to it, and unaffected by it, may be looked on as a jet striking the plane of the verge board, the angle between its direction and the normal to the plane of the verge board being equal to α'.

In Case III. it is clear that the problem cannot be solved without taking into account the relative area of the jet and the vane. Thus the area of the vane must be less than sufficient to cause the water to move parallel to its plane; and, if less, the angle of inclination of the verge boards to the normal R X must not be less than that of the jet after it leaves the first vane. If that inclination can be ascertained in terms of the relative areas of the jet and the vane, then the problem may be solved in the way pointed out in Case II., otherwise not. In Case II. the area of the vane is, of course, supposed to be sufficiently great to allow the water to spread quietly round the direction of the original jet parallel to the plane of initial impact.

Referring to Case I., if we put $\beta = O$, $\alpha = 90°$, that is, if the direction of the jet and the normal to the vane be horizontal, we get

$$R = \frac{\pi\ \omega\ v^2}{g} = 2\ \pi\ \omega \frac{v^2}{2\ g},$$

If the vane be pressed against the orifice, so that the flow ceases, the pressure upon the vane will be equal to $\pi\ \omega\ .\ \frac{v^2}{2\ g}$. From this Viry concludes that the statical pressure due to the jet is double the statical pressure of the head of water due to the velocity of the jet. It is, however, a case of comparing unlike things. R is not the instantaneous statical pressure caused by the jet, but the resistance overcome in an unit of time. In fact $R = \frac{\pi\ \omega\ v^2}{g} = \pi\ \omega\ v\ .\ \frac{v}{g}$, that is to say, R is equal to the statical measure of the impulse with which a body, whose mass is equal to that of the volume of water discharged in an unit of time, moving with a velocity v, would strike the vane.

Section II.—Description of Vortex Turbines erected at Reading by Messrs. Lawson and Mansergh for the Local Board of Health, and Analysis of Experiments made to test their Efficiency.

These turbines have been erected at the outfall pumping station of the Reading sewers at Blake's Lock on the Kennet, for the purpose of utilising the water power at that point purchased by the Urban Sanitary Authority. The head of water varies from 3 feet 9 inches to practically nothing in flood time, when the Thames water backs up to the level of the head water.

The minimum quantity of water available in the driest season is about 8,000 cubic feet a minute. The turbine-makers had to base their tender on the following conditions:—

(1.) To provide one or two turbines capable of utilising at least the minimum quantity, and a second or third to be used whenever there was sufficient water.

(2.) To state the effective power they were prepared to guarantee for a 3-feet head.

The tender of Messrs. Williamson Brothers, of Kendal, to erect three of Professor Thompson's vortex turbines, one with fixed and two with movable guide blades, was accepted. Fig. 3 is a horizontal section through the drum, with the guide blades at their standard opening of $8\frac{3}{4}$ inches. The internal diameter of the turbine is 4 feet 6 inches, and external 7 feet 6 inches. The depth of the guide-blade orifice is 2 feet $8\frac{1}{2}$ inches. That of the turbine is the same throughout, and equal to 2 feet 8 inches, being less than that of the orifice of supply by the thickness of a diaphragm which divides it into two equal portions.

The inner curve of the guide blade seems to be struck from two centres, o'', o' being the position of these centres when the guide blade is fixed at the standard opening. The sweep of the vanes consists of two reverse curves separated by an interval of straight. The centres of the exterior curves coincide with the point of intersection of the next but one preceding vane with the dotted circle. The tangent to the vane at the inner periphery passes through the point where the succeeding half-vane meets the exterior periphery, and the intervening straight touches the dotted circle shown close to inner periphery. The centres of the corresponding curves lie on the inner dotted circle.

If we suppose that the direction of the water between a and d runs in threads parallel to the outer guide blade, and to the inner guide blade between x and a, the angle α, at which the jet meets the tangent to the outer periphery, will be at x and d 8° 38′, at a 5° 43′, b 7°, 9′, and at c 8° 5′. It is therefore impossible with guide blades of this description to design a theoretically perfect turbine.

The angle β, at which the vanes cut the discharging side, is equal to 21° 10′, and the angle, at which a perpendicular to any vane through its extremity cuts the radius through the point where it intersects the preceding vane, is 22° 2′. The vanes cut the periphery on the receiving side at an angle of 65° 30′.

These data are taken from a sectional plan drawn to a scale of an inch to a foot, and may not therefore be exactly correct, but they are sufficiently near for the purpose of the present discussion. Assuming that the values above assigned for the angles at which the vanes cut the outer and inner periphery are correct, we may readily ascertain the corresponding theoretical value of α, if we leave friction out of consideration, and suppose that the turbine is always flowing full.

If w, v be the initial velocities of whirl of the outside periphery and the water, the whirling velocity of the inner periphery will be $\mu\, w$; and if the velocity of whirl of the water be destroyed, the final velocity of flow will be $\mu\, w \tan \beta$. Since the initial velocity of flow is $v \tan \alpha$, and the final area of flow μ times the initial area, we must have

$$\mu^2\, w \tan \beta = v \tan \alpha.$$

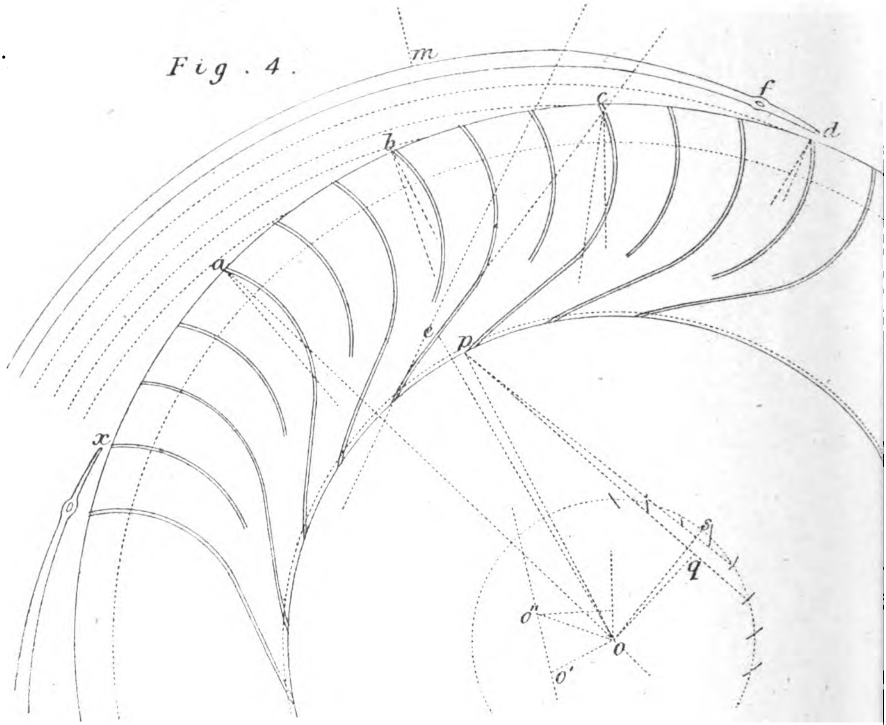
Also, in order that the water may strike the vanes without shock,

$$\frac{v \tan \alpha}{v - w} = \tan 65^\circ\, 30',$$

whence $\alpha = 7^\circ\, 31'$.

According to the maker's guarantee, the best velocity of rotation of the turbine for a 3-feet head is 24 revolutions per minute. For this head, therefore, $\mu w = 339 \cdot 3$ lineal feet per minute. Since the velocity of whirl is supposed to be wholly destroyed, the final relative velocity of flow at the point p will be equal to $\mu w \sec \beta = 363 \cdot 8$ lineal feet. At the point e the relative velocity cannot be less than $\frac{o\,e}{o\,p}\,\mu\,w \sec 22^\circ\,2' = \frac{2 \cdot 46}{2 \cdot 25}\,\mu\,w \sec 28^\circ\,2' = 400 \cdot 2$ lineal feet, since the sum of the areas of the openings perpendicular to the relative velocity at the inner periphery is equal to $24\,\{\,2$ feet 8 inches $\times\, 2\frac{1}{4}$ inches $\}\, = 12$ superficial feet, the discharge ought to be

Fig. 4.



Scale $\frac{3}{4}$ inch = 1 foot.

between 4,365 cubic feet and 4,800 cubic feet per minute. The initial velocity of flow is equal to $v \sec \alpha = \frac{\mu^2\,w \tan \beta}{\sin \alpha} = 605 \cdot 3$ lineal feet per minute, which is due to a head of 1 foot 7 inches. The sum of the areas of the guide blade openings is equal to $4\,\{\,2$ feet $8\frac{1}{2}$ inches $\times\, 8\frac{3}{4}$ inches $\}\, = 7 \cdot 9$ superficial feet, and therefore the corresponding discharge to 4,782 cubic feet per minute, which is equal to the maximum obtained by computing the discharge from the final relative velocity.

Conversely, we may ascertain what ought to be the initial velocity of the water, and the corresponding velocity of rotation of the turbines, in order that the volume of discharge may be equal to 4,000 cubic feet per minute. Since the guide-blade area is equal to 7·9 superficial feet, the initial velocity per minute ought to be 506·3 lineal feet, which is due to a head of 13¼ inches. According to Professor Thompson's theory, this ought to be equal to half the nett head. Therefore the nett would be equal to 26½ inches, which is only 74 per cent. of the whole head, 36 inches. Therefore the co-efficient of efficiency of the turbine must be less than ·74, since the loss of power due to frictional resistances of the moving parts in contact with the external water and the bearing is not included in the 26 per cent. Corresponding with this velocity of the water, the velocity of whirl of the outer periphery ought to be, in feet per minute,

$$w = \frac{506 \cdot 3 \, (\tan 65 \cdot 30 - \tan \alpha) \cos \alpha}{\tan 65 \cdot 30} = 472,$$

and therefore the correct number of revolutions 20. The quantity which the turbine would be capable of discharging per minute, estimated from the final relative velocity on the supposition that the final absolute velocity is radial, would be between 3,636 and 3,996 cubic feet per minute. According to the principles of construction laid down by Professor Thompson, the velocity of the external periphery ought to be equal to that of the whirling velocity of the water, and the vanes ought in consequence to cut that periphery at right angles. We should then have $w = 506 \cdot 3 \cos \alpha = 502$ lineal feet, and the number of revolutions 21·3 per minute. The discharge estimated from the last data would then lie between 3,876 and 4,260 cubic feet per minute.

Whether or not the correct discharge of the turbine exceeds 4,000 cubic feet, it is clear that it will not pass the same quantity as the guide-blade orifices with a velocity due to 74 per cent. of the whole head. There must, therefore, exist within the guide blades and in some parts of the turbine a pressure greater than that due to the tail water and the atmosphere.

According to Professor Thompson's theory, this is due to the action of a centrifugal force generated by the rotation. It is stated in his papers that owing to the action of this centrifugal force the motion of the turbine is kept steady. Thus, if going too slow, the intensity of the force diminishes and the flow increases; if going too fast, the intensity increases and the flow diminishes. Clearly, therefore, Professor Thompson supposes that the water has the same angular velocity of rotation as the turbine. Such, however, is never the case. When the turbine is moving at its best speed, the issuing water has no velocity of rotation round the turbine shaft; and if it be moving at less than its best speed, the direction of the tangential component of the final absolute velocity of the water is actually opposite to that of the turbine.

The only centrifugal force which can possibly exist must be due to the curvature of the absolute path, which will always be directed from the centre of curvature of each point of that path, and, being at right angles to the direction of motion, can never retard the velocity of flow.

If, then, we reject the theory of Professor Thompson, the preceding investigations afford another explanation of the existence of this pressure, viz. the incorrect relative proportions of the orifices of entry and discharge.

Let the turbine be reduced to rest by impressing upon it and the water velocities equal and opposite to the actual velocities of each part. This will not affect the quantity of discharge. In accordance with Bernouilli's theorem, the sum of the heads due to the pressure and velocity at each point must be constant, if friction be left out of consideration. If friction be taken into consideration, the sum of the heads due to the velocity and pressure at the orifice of discharge from the guide blades is equal to the sum of the heads due to the initial pressure and the initial velocity of flow on the receiving side of the turbine, to the friction in passing through the turbine, to the tail water and the atmosphere. The two last are common to both points. Therefore the head due to the relative velocity of discharge must be equal to the head due to the initial relative velocity, plus the head due to initial pressure, minus the heads due to the atmosphere, tail water, and friction, and we get the following equation:

$$(1)\ \frac{u^2}{2g} = (1 - K)\, c\, H \sin^2 \alpha\, (1 + \cot^2 65\cdot 30) + K\, c\, H,$$

in which u is equal to the final relative velocity, c H to the nett initial head, and K c H the nett initial head due to pressure. There are forty-eight vanes externally, each ¼ inch thick. The external area of flow is therefore equal to 59·96 square feet. Since the supply must equal the demand, we have

$$(2)\ 12\, u = 59\cdot 96 \sin \alpha \sqrt{2\, g\, (1 - K)\, c\, H}.$$

Combining these two equations, we get K = ·29. Now the area of the guide-blade orifice must be such that the velocity of flow through it is that due to the nett head $\sqrt{2\, g\, (1 - K)\, c\, H}$. Hence, if A be that area, we must have

$$A \sqrt{2\, g\, (1 - K)\, c\, H} = 12\, u$$
$$= 59\cdot 96 \sin \alpha \sqrt{2\, g\, (1 - K)\, c\, H},$$

whence A = 7·83 square feet. The actual area at the standard opening is 7·9 square feet.

The value of c depends on the magnitude of the internal diameter of the turbine and the area of the tail race. Professor Thompson appears to have adopted the value ·74. The initial velocity would then be due to ·53 H. When H is equal to 3 feet, the initial velocity of the water in feet per minute would be 606 lineal feet, and the corresponding velocity of whirl of the outer periphery of the turbine and the number of rotations per minute would be 565 lineal feet and 24 respectively. The discharge in cubic feet per minute would be 4,787.

The following experiments have been made to test the efficiency of the turbines. (See next page).

The difference in the maximum deficiencies, about 2 per cent., obtained on October 5th and November 29th, may possibly be due to a variation in the pump efficiency. The still greater difference which exists between those obtained on October 31st and November 5th, an interval of only six days, the Author attributes to the newness of the machinery and the more unsteady working of the turbines due to the following cause. Connected with the pump wells there is an overflow to the river to provide against all emergencies. On the first day the sluice was opened to produce an equality in the height of the lift. This caused the pumps to be buried, so that the rams had very little work to do during the up-stroke.

Experiments Nos. 22 and 24 were made with the guide blades opened 3½ inches beyond the standard.

The quantities of discharge and of water lifted in the above columns have been estimated on the assumption that the co-efficient of discharge through the guide-blade orifices is unity, and the pump efficiency perfect.

On November 29th a series of seven experiments was made to test the pump efficiency, by carefully noting the number of strokes made whilst the water descended a depth carefully measured in the pump wells. The least number of strokes made in a single experiment was 68, equivalent to 272 strokes of a single pump. The co-efficients obtained varied from ·76 to ·79, the mean being ·77.

TABLE I.

Number of Experiment.	Head of Water in feet.	Lift of Pumps in feet.	Length of Stroke. ft.	Length of Stroke. ins.	Number of Strokes per minute.	Velocity of Periphery / $\sqrt{2gH}$	Foot lbs. of Water lifted to top of Main.	Foot lbs. of Water discharged through Turbines per minute.	Corresponding Efficiency of Pumping Power.
EXPERIMENTS MADE OCTOBER 31ST:—									
1	3·35	38	2	6	7·7	·50	320·037	1,456,838	·220
2	3·32	37·1	2	5	8·5	·54	333·416	1,438,322	·231
3	3·43	36·5	2	4	9·9	·62	368·220	1,505,000	·244
4	3·30	36·4	2	8	9·6	·61	343·980	1,421,000	·242
5	3·24	36·4	2	4	9·1	·59	338·136	1,390,848	·243
6	3·13	36·4	2	4	7·7	·52	286·113	1,311,477	·218
7	3·23	36·7	2	2	10·3	·67	358·321	1,376,100	·260
8	3·10	36·7	2	1	9·8	·63	327·815	1,281,610	·256
9	3·42	36·8	2	0	11·5	·72	370·296	1,499,000	·246
10	3·36	36·8	1	11	11·0	·70	339·441	1,456,838	·233
EXPERIMENTS MADE NOVEMBER 5TH:—									
11	3·43	43·6	2	6	9·0	·57	429·400	1,505,000	·285
12	3·51	44·4	2	5	10·0	·63	469·700	1,559,000	·300
13	3·57	43·5	2	4	11·0	·68	488·700	1,599,264	·305
14	3·53	43·6	2	3	11·3	·71	485·200	1,572,300	·309
15	3·48	43·5	2	2	11·0	·69	453·800	1,539,200	·295
16	3·56	43·5	2	1	11·3	·70	448·120	1,592,500	·281
17	3·74	45·0	2	4	11·5	·69	528·523	1,714,600	·308
18	3·22	44·8	2	4	9·3	·61	425·410	1,369,800	·310
19	3·40	43·1	2	0	11·0	·70	415·030	1,486,200	·279
20	3·08	43·1	2	0	10·2	·68	384·760	1,281,610	·300
21	2·41	43·1	2	0	7·0	·53	264·050	886,920	·300
22	3·17	43·2	2	4	10·3	·68	454·530	..	..
23	3·30	43·2	2	4	9·5	·62	419·230	1,421,000	·295
24	3·31	43·3	2	4	10·7	·69	473·070	..	..
EXPERIMENTS MADE NOVEMBER 29TH:—									
25	2·8	42·3	2	3	8·7	·61	362·420	1,110,700	·326
26	2·8	42·4	2	2	8·9	·63	363·377	1,110,700	·327
27	2·7	42·4	2	1	9 5	·67	370·241	1,110,700	·333

The Author has shown that, if the theory maintained by him be correct, the best ratio of the speed of the receiving side of the turbine to the velocity of the water due to the whole head, represented in the corresponding column by $\sqrt{2gH}$, is ·8[illegible], when the nett head is equal to 74 per cent. of the whole head. Since the loss due to friction in this case has nothing to do with the friction of the external water, but only with the internal friction, 24 per cent. seems a large deduction, since the converging sides of the guide-blade orifices prevent any contraction. If we compare the values of this ratio given in the Table, we find that the values which correspond with the best efficiency lie between ·6 and ·7. The mean of those obtained on November 29th, on which the co-efficients of efficiency were greater than any obtained before, being ·64. The corresponding value of c would be $\frac{\cdot 74 + \cdot 64}{\cdot 53} = \cdot 84$, and the discharge 5,100 cubic feet.

The actual discharge has been approximately arrived at in the following manner. A plank weir was built in the tail race, the crest of which came up within about 2 feet of the surface. To the edge of the crest a trough, 4 feet 6 inches long and about 2 feet 6 inches deep, was fixed, with its bottom as nearly horizontal as possible. The sides of this trough were flush with the sides of the walls of the tail race. With this contrivance the area of flow could always be ascertained with great exactness, the length of the weir being 16 feet 6 inches.

The speed of the current was ascertained by means of a current meter purchased of Messrs. Elliot. On account of the discrepancies observed during the four first experiments, the duration of each of the subsequent experiments was carefully noted, in order to see if these discrepancies were owing to variation in the rate at which the current meter traversed the water. The extreme rates do not differ much from the extremes obtained during the gauging of the velocity of the tail race.

Number of Experiment.	Duration of Experiment in minutes.	Length in lineal feet registered.	Actual Length in lineal feet traversed.	Percentage to be added to registered quantity.
1	..	1,232	1,320	7·1
2	..	1,189	1,320	11·0
3	..	1,289	1,320	2·4
4	..	1,264	1,320	4·4
5	20	1,212	1,320	·8·9
6	8	1,301	1,320	1·4
7	11	1,274	1,320	3·6
8	7	1,299	1,320	1·6
Average percentage 5·0.				

The velocities of the current were taken at the different points described in Table II., in a section 2 feet 3 inches from the inner face of the weir. Those taken at 18 inches above the bottom ought strictly to have been taken at 16 inches, but the Author did not at starting anticipate such a marked difference in the rate at the various points. This difference was due to the current of water discharged from the under side of the turbine, which rises up vertically against the weir, and causes eddies and cross currents even at the distance of 2 feet 3 inches from the inner edge of the crest. In order to get a reliable gauging, it was necessary on this account to take the velocities at a great number of points.

TABLE II.

No. of Experiment.	Height above bottom of Weir Trough.	Distance from the Side.	Duration of Experiment.	Head above Still Water in adjoining Bay.	Head above Surface of Water at Section.	Head above Surface of Water over inner edge of Drum.	Number of Revolutions.	Depth over Weir.	Lineal feet registered by Meter.	Velocity in lineal feet per minute reduced to 3-feet Head.	Discharge in cubic feet per minute according to Meter.	Discharge in cubic feet per minute corrected for error in rate.
	Ins.	Ft. Ins.	Min.									
1	8	1 6	15	3·57	3·57	3·31	23½	1·53	1,310	80		
2	8	8 0	15	3·37	3 36	3·13	23¼	1·77	1,988	125		
3	8	4 0	15	3·24	3·25	3·00	22	1·74	2,326	149		
4	8	1 6	15	3·28	3·22	3·08	22½	1·90	1,210	77		
5	12	1 6	15	3·21	3·18	2·99	21½	1·90	2,568	166		
6	12	4 0	15	3·16	3·13	2·96	21½	1·90	2,388	155		
7	12	8 0	15	3·12	3·09	2·97	21	1·90	2,628	172		
8	18	8 0	15	3·08	3 05	2·94	20¼	1·90	2,850	188		
9	18	4 0	15	3·12	3·09	3·04	21¾	1·90	2,496	163		
10	18	1 6	15	3·17	3·14	3·01	21¾	1·90	2,508	163		
11	18	6 0	15	3·13	3 11	2·98	21¾	1·86	2,825	184		
12	12	6 0	15	3·19	3 18	3·02	22½	1·95	2,358	153		
13	8	6 0	15	3·02	3·02	2·81	21	1·96	737	49		
14	20	8 0	15	3·11	3·13	2·96	22¼	1·90	2,510	165		
15	20	6 0	15	3·05	3·07	2·90	21½	1·97	2,708	179		
16	20	4 0	15	2·97	2·98	2·84	20½	1·98	2,556	171		
17	20	1 6	15	3·09	3·09	2 89	21½	1·91	2,530	167		
18	4	8 0	15	3·09	3·12	2·88	21½	1·87	990	65		
19	4	6 0	15	3·06	3 10	2·90	21¼	1·91	22	1½		
20	4	4 0	15	3·07	3·09	2·85	20¾	1·91	1,103	73		
21	4	1 6	15	3·13	3·16	2·96	21¼	1·85	706	46		
22	20	1 0	10	2·84	2·83	2·75	21¾	2·12	909	94		
23	20	3 0	10	2·82	2·80	2·69	22	2·09	1,385	142		
24	20	5 0	10	3·05	3·02	2·91	23¼	1·99	1,576	157		
25	20	7 0	10	3·02	2·97	2·83	21½	2 08	1,709	171		
26	18	1 0	10	2·89	2·88	2·80	21	2·12	1,573	160		
27	18	3 0	10	2·89	2·88	2·79	21	2·12	1,459	149		
28	18	5 0	10	2·89	2·88	2·78	21	2·12	1,338	136		
29	18	7 0	10	2·89	2·88	2·77	21	2·12	1,450	148		
30	12	1 0	10	2·89	2·88	2·77	21	2·12	1,430	146		
31	12	3 0	10	2·91	2·90	2·82	21¾	2·12	1,484	151		
32	12	5 0	10	2·97	2 96	2·89	21¼	2·12	1,288	131		
33	12	7 0	10	3·02	3·01	2·87	21¼	2·12	1,846	184		
34	8	1 0	10	2·79	2·81	2·66	20½	2·31	711	74		
35	8	3 0	10	2·72	2·78	2·59	20	2·26	1,033	109		
36	8	5 0	10	2·88	2·87	2·71	20½	2·12	868	87		
37	8	7 0	10	2·88	2·87	2·72	20¼	2·12	1,206	123		
38	4	1 0	10	2·85	2·84	2·69	20	2·12	312	32		
39	4	3 0	10	2·92	2·90	2·82	20½	2·08	666	67		
40	4	5 0	5	2·92	2·90	2·80	21	2·08	372	75		
41	4	7 0	5	2·94	2·92	2·81	20¾	2·04	146	29		
	Average values.							2·01	..	124·6	4,135	4,341

Up to No. 21, inclusive, the distances are measured from the north side of the tail race; from No. 22 to No. 41, inclusive, from the south side. The current on the south side ran very steady, and the meter was not turned from side to side by the eddies. This difference between the currents on the two sides may probably be owing to the fact that the turbine rotates from north to south when the observer stands with his face towards the east in the middle of the tail race, or the turbine itself may have been working more steadily during the second day's experiments. When the turbine is not running at its best speed there is a great commotion in the issuing water.

The water is let on to the turbines by three sluices. In order to check the gaugings given in Table II., two of the three sluices were closed at the conclusion of the experiments, and the difference in level between still water above and below the sluice carefully observed. It was found to be 2 inches, the gross head being 2·77 feet. The width between the vertical frames is 5 feet 4 inches; between the edges of the grooves in which the sluices run, 5 feet. The vena contracta, however, lay wholly within the edges of the grooves. We must therefore apply the co-efficients of the discharge to the outer area, which is equal to 34·2 square feet. The bottom of the sluice bay is protected in front by an horizontal apron, which is only 2 inches below the level of the sluice sill.

The respective discharges of 4,341 cubic feet, ascertained by experiment, and the calculated discharge of 4,787 cubic feet, and 5,100 cubic feet obtained on the supposition that the construction of the turbine is theoretically perfect, and without taking any account of internal friction, become 4,171 cubic feet, 4,600 cubic feet, and 4,900 cubic feet for a 2·77-feet head. The corresponding co-efficients of discharge through the sluice would be ·62, ·68, and ·72 respectively. The first is the smallest adopted for gauging the discharge through a small orifice in a thin plate at a distance from top, bottom, and sides, and is therefore clearly too small in this instance. The last, ·72, cannot be looked upon as too large for computing the discharge through a notch 5 feet 4 inches by 6 feet 5 inches, towards which the water moves along a horizontal platform almost flush with the bottom of the notch.

The pumping machinery consists of four ram pumps, driven by spur and pinion gearing, which reduces the speed of the two pump shafts. The horizontal shaft revolves at the same rate as the turbines, and is actuated by the turbines by means of mitre wheels. The above experiments give the efficiency of the pumping machinery as a whole, including the friction in the rising main, the internal diameter of which is 2 feet. The velocity of flow in the rising main during the experiments would never reach 10 inches a second.

For a discharge of 4,341 cubic feet the efficiency of the machinery would be ·41, of 4,787 cubic feet ·35, and of 5,100 cubic feet ·33, if we take ·33, the average of the three last experiments in Table I., as our basis. In the following Table the first row of figures gives the co-efficient of the work done in overcoming the friction of the pumps, rising main, and gearing, in terms of the foot pounds of water lifted, and the remaining columns the corresponding efficiency of the turbines for the three different discharges.

..	·15	·20	·30	·40	Mean Values.
4,341	·44	·46	·49	·53	·48
4,787	·40	·42	·46	·49	·44
5,100	·38	·40	·43	·46	·42

LONDON: PRINTED BY WILLIAM CLOWES AND SONS, STAMFORD STREET
AND CHARING CROSS.

CATALOGUE OF BOOKS.

Hydraulics.

Practical Hydraulics: a Series of Rules and Tables for the use of Engineers, &c, &c. By THOMAS BOX. Fourth Edition. *Numerous Plates.* Post 8vo., cloth, 5*s*.

CONTENTS:

Tables and Formulæ of the Theoretical Discharge of Water by round apertures of various diameters, and under different heads of water pressure.

Of the Actual Discharge of short Tubes of various diameters, with square edges, and under different heads of water pressure, being 8-10ths of the theoretical discharge For bends in Water pipes, showing the loss of head due to change of direction by one bend of 90°. For quick bends.

Of the Heights of Jets with different heads.

Of the Discharge of Jets with different heads.

Of the Discharge of Water over Weirs.

Of the Discharge of Overflow Pipes for Tanks, &c. For open Channels, Canals, and Rivers. For the Discharge of ditto.

Of the Strength of Water Pipes.

&c. &c. &c.

Hydraulics.

Manual of Hydrology, containing Hydraulic and other Tables, rivers, flow of water, springs, wells, and percolation, tides, estuaries and tidal rivers, rainfall, and evaporation. By NATHANIEL BEARDMORE, C.E. *Plates.* 8vo., cloth, 24*s*.

Hydraulics.

Lowell Hydraulic Experiments; being a Selection from Experiments on Hydraulic Motors, on the flow of Water over Weirs, in open Canals, of uniform Rectangular Sections, and through Submerged Orifices and Diverging Tubes, made at Lowell, Massachusetts. By JAMES B. FRANCIS, Civil Engineer. Second Edition, revised and enlarged, with many New Experiments and Additional Illustrations. *Plates.* 4to., cloth, £3 13*s*. 6*d*.

Hydraulic Engineering.

Prize Essay on the Encroachment of the Sea between the River Mersey and the Bristol Channel, being an Essay which obtained the Prize of the National Eistedfodd, held at Chester, 1866. By J. E. THOMAS. 8vo., sewed, 1*s*.

Hydraulics of Great Rivers.

Observations and Surveys on the Largest Rivers of the World. By J. J. RÉVY, Civil Engineer. One volume, imperial 4to, cloth, with *eight large Plates and Charts*, £2 2*s*.

Hydraulic Tables.

Office Hydraulic Tables, for the use of Engineers engaged in Waterworks, giving the Discharge and Dimensions of Rivers, Channels, and Pipes. By J. NEVILLE. On a large folio sheet, 1*s*.

Engineers' Pocket-Book.

A Pocket-Book of Useful Formulæ and Memoranda for Civil and Mechanical Engineers. By GUILDFORD L. MOLESWORTH, Mem. Ins. C. E., Consulting Engineer to the Government of India for State Railways. Eighteenth Edition, revised, with considerable additions by the Author; together with a valuable contribution on Telegraphs, by R. S. BROUGH. 32mo., roan, 6*s*.

Ditto, Interleaved with ruled Paper for Office use. 9*s*.

Ditto, printed on India paper, for the waistcoat pocket, 6*s*.

SYNOPSIS OF CONTENTS.

Surveying, Levelling, &c.
Strength and Weight of Materials.
Earthwork, Brickwork, Masonry, Arches, &c.
Struts, Columns, Beams, and Trusses.
Flooring, Roofing, and Roof Trusses.
Girders, Bridges, &c.
Railways and Roads.
Hydraulic Formulæ.
Canals, Sewers, Waterworks, Docks.
Irrigation and Breakwaters.
Gas, Ventilation, and Warming.
Heat, Light, Colour, and Sound.
Gravity—Centres, Forces, and Powers.
Millwork, Teeth of Wheels, Shafting, &c.
Workshop Recipes.
Sundry Machinery.
Animal Power.
Steam and the Steam Engine.
Water-power, Water-wheels, Turbines, &c.
Wind and Windmills.
Steam Navigation, Ship-Building, Tonnage, &c.
Gunnery, Projectiles, &c.
Weights, Measures, and Money.
Trigonometry, Conic Sections, and Curves.
Telegraph.
Mensuration.
Tables of Areas and Circumference, and Arcs of Circles.
Logarithms, Square and Cube Roots, Powers.
Reciprocals, &c.
Useful Numbers.
Differential and Integral Calculus.
Algebraic Signs.
Telegraphic Construction and Formulæ.

"Most of our readers are already acquainted with Molesworth's pocket-book, and not a few, we imagine, are indebted to it for valuable information, or for refreshers of the memory. The book has been re-arranged, the supplemental formulæ and tables added since the first issue, having now been incorporated with the body of the book in their proper positions, the whole making a handy size for the pocket. Every care has been taken to ensure correctness, both clerically and typographically, and the book is an indispensable *vade mecum* for the mechanic and the professional man."—*English Mechanic.*

Turbine.

A Practical Treatise on the Construction of Horizontal and Vertical Water-wheels, *with* 11 *Plates*, specially designed for the use of operative mechanics. By WILLIAM CULLEN, Millwright and Engineer. Second Edition, revised and enlarged. Small 4to., cloth, 12*s*. 6*d*.